Financial Reforms and Consumption Smoothing

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Abstract
We study how financial reforms or liberalization affect the extent of consumption smoothing in a dynamic stochastic general equilibrium model of an emerging economy. The empirical literature highlights that financial reforms and development improve consumption smoothing in industrialized economies, but not in emerging economies. Our model emphasizes consumer credit and durable purchases via home production. In accord with the empirical literature, our model predicts that equity market reforms ameliorate consumption smoothing. Our model also predicts that reforms that relax interest and credit controls may deteriorate consumption smoothing in emerging economies where a large share of the population is credit constrained and where home production is particularly important.

Keywords: Home production, collateral constraint, durable, and consumption volatility.
JEL Classifications: F41; F44

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1 Introduction

We study how financial reforms affect the extent of consumption smoothing in a dynamic stochastic general equilibrium model of an emerging economy. In contrast to most of the literature, we focus our attention on domestic reforms and their effects on household credit, durable purchases, and consumption smoothing.

Our interest in this issue is motivated by the changes in the relative volatility of consumption in emerging economies during the 1990's wave of reforms. To focus our attention, Figure 1 plots the average relative volatility of consumption for a group of small industrialized economies and a group of emerging economies over time. The data is described in the Data Appendix. The relative volatility of consumption refers to the ratio of the standard deviation of the cyclical component of consumption to the standard deviation of the cyclical component of gross domestic product, where the cyclical component is extracted by a HP filter with a smoothing parameter of 100 (see Hodrick and Prescott, 1997). The figure shows the cross-sectional averages using a 20-year rolling window. A look at the figure suggests that consumption smoothing in emerging economies differs in two important ways from consumption smoothing in small industrialized economies. First, consumers in emerging economies appear unable to smooth consumption. That is, on average, consumption is more volatile than output in emerging economies. Second, the extent of consumption smoothing in emerging economies appears to deteriorate in the early 1990s. To be clear, the average relative volatility of consumption in emerging economies rose precisely during the early 1990's wave of financial reforms (see Abiad, Detragiache, and Tressel, 2010).

In theory, the ability of consumers to smooth consumption should depend on access to developed financial markets. The cross-sectional evidence in Figure 1 is consistent with the hypothesis that greater access to financial markets helps consumer smooth consumption. For example, consumers in small industrialized economies where financial markets are more developed appear to smooth consumption well. Consumers in emerging economies, where financial markets are less developed, appear unable to smooth consumption. As a result, aggregate consumption tends to be more volatile than output in emerging economies. The logic should extend in the time series as well. That is, consumers in emerging economies that are granted better access to financial markets after financial reforms are

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1 The small industrialized economies in panel (a) include Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, Norway, and Sweden. The emerging economies in panel (b) are Argentina, Brazil, Chile, Colombia, Hong Kong, Indonesia, Korea (Republic Of), Malaysia, Mexico, Peru, Philippines, Thailand, Venezuela, and South Africa.

2 This observation is the subject of a growing body of literature (see Aguiar and Gopinath, 2007; Chang and Fernandez, 2013; Garcia-Cicco, Pancrazi, and Uribe, 2010; and Neumeyer and Perri, 2005).
reforms should smooth consumption better. The reforms should lower the relative volatility of consumption. The data however suggests that this is not the case.

To study these issues, we develop a closed economy model that emphasizes important determinants of consumption behavior from the empirical literature that studies consumption smoothing in developing countries using micro data. In particular, both Aron and Muellbauer (2013) and Rosenzweig and Wolpin (1993) argue that credit constraints and the accumulation of durable physical assets are important determinants of consumption behavior. In doing so, we depart from the literature that ignores these elements and mostly focuses on international reforms and globalization. Examples include Bai and Zhang (2012), Bhattacharya and Patnaik (2015), Faia (2011), Leblecioglu (2009), and Levchenko (2005).

In our closed economy model, the consumer credit constraint is modeled as a collateral constraint, as in Kiyotaki and Moore (1997). In contrast to Kiyotaki and Moore (1997), we emphasize the amplification of productivity shocks on consumption fluctuations rather than on output fluctuations. In addition, the collateral asset is durable consumption or, similarly, the home capital used in household production rather than market capital. Our modeling of home production follows the work of Benhabib, Rogerson, and Wright (1991) and Parente, Rogerson, and Wright (2000). In our context, home production is important for a few reasons. First, home production is likely more important in emerging economies than in industrial economies (see Parente, Rogerson, and Wright, 2000). Also, home production provides a motive for households to accumulate durable goods, home capital, that can serve as a collateral. Finally, home production serves as a source of idiosyncratic consumer risk.

The general layout of our model is as follows. The closed economy is populated by households that consume market and home produced goods. The market good is produced with market capital and labor. Labor is supplied by all households. The non-market or home goods are also produced

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3 Other relevant studies include Karlan, Osei, Osei-Akoto, and Udry (2014) and Kazianga and Udry (2006). There is also an important literature that links consumption behavior to credit constraints and durables in industrialized economies. Examples include Alessie, Devereux, and Weber (1997), Attanazio, Goldberg, and Kyriazidou (2008), and Chah, Ramey, and Starr (1995).

4 Among these, Bhattacharya and Patnaik (2015) studies financial inclusion in a model with hand-to-mouth consumers, but ignores the ability of poor households to save in durables. We also acknowledge a much larger literature that studies financial development and volatility, including Aghion, Bacchetta, and Banerjee (2004), Iyigun and Owen (2004), and Wang, Wen, and Xu (2016). Unsurprisingly, the empirical literature finds that the response of consumption smoothing to international liberalization and globalization in emerging economies differs from the response in industrialized economies. For example, Kose, Prasad, and Terrones (2003) find that financial liberalization helps improve the extent of risk sharing and consumption smoothing for industrialized economies, but not for emerging economies.

5 Gomme and Zhao (2010) and Restrepo-Echevarria (2014) study the importance of home production, and more generally of the informal economy, on the volatility of consumption. These authors however do not consider the interplay with financial constraints and financial reforms.
with capital and labor. In this context, the stock of home capital is the stock of durable accumulated by households. There are two types of households. Some households, the unconstrained households, are patient and enjoy unconstrained access to both equity and debt markets. Other households, the constrained households, are impatient and have only restricted access to financial markets. These households do not have access to the equity market. Their abilities to borrow on the debt market is also limited by a collateral constraint, where their only collateral asset is their stock of durable goods.

We analyze the model using numerical simulations. The simulations reveal that financial reforms that grant access to the equity market to a larger fraction of households unambiguously reduce the relative volatility of consumption and ameliorate consumption smoothing. This seems to match well with the empirical findings about equity market liberalization found in Bekaert, Harvey, and Lundblad (2006). The simulations also reveal that financial reforms that relax the constrained households’ collateral constraint may raise or reduce the relative volatility of consumption. To be precise, the simulations show that a relaxation of the collateral constraint produces a hump shaped response in the relative volatility of consumption. Importantly, when the households are highly constrained, a relaxation of the constraint raises the volatility of durable purchases, the collateral asset. This translates to the relative volatility of aggregate consumption as long as a large share of the population faces the collateral constraint and has no access to the equity market. In addition, these effects are quantitatively more important when home production itself is more important. This suggests that a deterioration of consumption smoothing following reforms is more likely in emerging economies, where equity markets are less developed and where home production plays a larger role than in industrial economies.

The economies portrayed in Figure 1 however can all be characterized as small open economies. These economies experience fluctuations in their current account and can thus vary their international borrowing. This latter ability might help smooth consumption and thus mitigate the effects of domestic financial reforms on consumption smoothing. In addition, Kaminsky and Reinhart (1999) and Ranciere, Tornell, and Westermann (2006) suggest that countries experience a higher propensity to crisis following financial liberalizations, and this should also affect consumption smoothing. It would then appear important to verify the robustness and importance of our results to international liberalization. For this reason, we extend our analysis to a small open economy setting. To do so, we close the small open economy by imposing a real interest rate rule in the spirit of Neumeyer and Perri (2005). This extension ensures countercyclical interest rates, as
generally observed in emerging economies (see Uribe and Yue, 2006). Numerical simulations show that our results on the effects of domestic financial reforms are robust to this extension, while also predicting high relative volatility of consumption.

2 Case Studies

Our novel interpretation of the effects of financial reforms relies on the behavior of durable consumption expenditures. Unfortunately, very few countries report durable consumption expenditures for a long enough sample that covers a sizeable financial reform. Nevertheless, both Korea and South Africa provide long samples that overlap major financial reforms. Thus, in lieu of a formal empirical analysis, we offer two case studies.

Our aim here is twofold. One, we highlight that other researchers have found that recent financial reforms in Korea and South Africa can be interpreted as an important relaxation of a collateral constraint faced by consumers. Second, we document that the volatility of consumption relative to the volatility of output is higher after reforms. Finally, we also provide some context to understand the reforms.

We are interested in the financial deregulation that occurred in the early to mid 1990s in both Korea and South Africa. In both cases, the deregulations followed pressures from the US and the international community. They included both domestic and international components. In Korea, the financial deregulation followed a three-phase plan that was to be implemented between 1993 and 1997. In South Africa, the financial deregulation came with the dismantlement of apartheid that occurred between 1990 and 1996.

At the same time, Korea and South Africa differ greatly. In the mid 1990s, Korea had low unemployment and low income inequality, while South Africa had high unemployment and high income inequality. For Korea, Kang (2001) reports a 1993 unemployment rate of roughly 3 percent and a Gini coefficient of roughly 30 percent. For South Africa, Aron and Muellbauer (2000) report a 1993 broad unemployment rate somewhere between 30 to 40 percent and a Gini coefficient of more than 60 percent. Another important difference is the extremely poor access to banks and credits in early 1990s South Africa. Ludwig (2008) reports that 60 percent of adults did not have access to banks or credits in 1994, while Okurut (2006) estimates that almost 90 percent of households did not have access to credits in 1995.

\footnote{This data restriction is well known. For example, the data for emerging economies in Alvarez-Parra, Brandao-Marques, and Toledo (2013) generally starts in mid to late 1990s.}
2.1 Korea

We start our case study by briefly reviewing Korea’s effort to deregulate financial markets starting in 1980. Park (1996) provides an overview of Korea’s experience with financial deregulation. For a broader perspective, Bekaert and Harvey (2005) provide a detailed chronology of financial deregulations and other important economic events, while Soon (1995) provides a narrative of the different reforms.

In the 1960s and 1970s, Korea’s government supported its export-led growth policy by directly intervening in financial markets. In particular, the government would issue lending directives to redirect credit toward exporting industries, while charging below market lending rates. Eventually, overinvestment in particular industries and an episode of stagflation in the late 1970s prompted a reevaluation of Korea’s economic policies. The new policy package included financial market reforms.

Korea’s deregulation of domestic financial markets was very gradual. The government deregulated lending rates progressively over the 1980s but some of these were reversed because of economic slowdowns at the end of the 1980s. In 1993, Korea announced a three-phase blueprint for financial deregulation. The blueprint called for both domestic and international liberalization. Importantly, the first phase (1993-94) involved a deregulation of all bank and nonbank lending rates, while the latter two phases called for further domestic and then international liberalization. Unfortunately, deregulation efforts in Korea were followed by a massive foreign exchange crisis in 1997. The crisis affected international borrowing, and this would have worsened the ability of consumers to smooth consumption.

Deregulation efforts were also followed by large changes in household debt. Park (2009) reviews changes in household debt for the 1998-2002 period that just followed the crisis. Interestingly, as predicted by our model, households in the lower quintiles of the income distribution saw large increase in household debt, while households in the upper quintiles saw no change or even a reduction in household debt. In particular, households in the second lowest quintile (21 to 40 percent) and third lowest quintile (41 to 60 percent) of the income distribution experienced a 41 percent and 32 percent increase in their average household debt. Households in the second top quintile (61 to 80 percent) did not experience much change in their household debt, while households in the top quintile (81 to 100 percent) experienced a seven percent drop in average household debt. Park (2009, p.169) argues that this is indicative of a relaxation of the collateral constraint for poor households:
“First of all, we can argue that the results provide indirect evidence for alleviation of credit constraints in the consumer credit market. *The fact that lower-income households experienced faster debt accumulation may imply the alleviation of severe liquidity constraint placed on them under the practices prevailing in the financial market before the economic crisis.* Before the economic crisis, direction intervention of the government in credit allocation was a common practice. The Korean government pursued the development policy to channel a disproportionately large amount of credit resources into a small group of targeted industries to promote faster growth. *It was not rare that households were not able to borrow even though they did possess enough assets to offer as collateral in some cases, let alone borrowing without collateral.* After the economic crisis in 1997, the Korean government gave up the traditional interventionist approach and let the market determine resource allocation in the credit market. It was then possible for financial institutions to increase the credit supply to the household sector with less concern about non-entrepreneurial factors.”

2.2 South Africa


Deregulation efforts in South Africa have some similarities with the efforts in Korea. For example, in the 1960s and 1970s, South Africa pursued government-led development and intervened heavily in financial markets. Eventually, international political pressures and rising inflation, as well as pressures on the South African rand, prompted a reevaluation of South Africa’s financial and monetary policies.

This reevaluation appeared in the reports of the Commission of Inquiry into the Monetary System and Monetary Policy of South Africa (the de Kock Reports) that appeared as interim reports in 1978 and 1982, and a final report in 1985. Amongst other policy recommendations, the reports advocate a deregulation of financial markets. As discussed in Aron and Muellbauer (2013) and Muyambiri and Odhiambo (2014), South Africa removed its interest and credit controls, as well as the implemented other financial reforms, in the early 1980s. Broader financial reforms were slowly instituted over time.

For our purpose, we are interested in the dismantling of apartheid that occurred between 1990
and 1996. Of particular interest is the Abolition of Racially Based Land Measures Act of 1991 that abolished restrictions on land ownership. This act is important because race-based land restrictions effectively eliminated the possibility of non-white consumers to pledge land as a collateral. Aron and Muellbauer (2013) study the importance of the collateral effect of housing wealth and financial reforms on consumption in South Africa. On collateral, they argue that, in the early 1990s, pensions were the main collateral in housing loans, while in the mid 1990s special mortgage accounts allowed consumers to borrow using the value of housing as the collateral. Aron and Muellbauer (2013, p. S193) conclude:

“Estimates from this model on aggregate data when there is certainly great heterogeneity of behavior at the micro-level need to be interpreted with care. For example, the estimated housing collateral effect after credit market liberalization for South Africa is estimated to be about twice or more as high as for the three Anglo-Saxon economies. The estimated effect is an average for a population with one of the highest levels of income inequality in the world and necessarily reflects a diverse set of micro-responses, zero for most households. It is plausible that the segments of the population where the responses are largest have been increasing their share of income and consumption. The growth of a Black South African middle-class, with low saving deposits but improving employment opportunities and confident expectations in future income, has likely led to an increase in spending linked to easier credit and higher collateral values, accounting for the large collateral effect. However, as noted above, the AIDS epidemic may well have caused a partial reversal of these tendencies from the late 1990s.”

Aron and Muellbauer (2013) acknowledge that some of the financial deregulation that accompanied the dismantlement of apartheid can be interpreted as a loosening of a collateral constraint on consumers. They also acknowledge however that few households had access to credits (a large fraction were unbanked). It may also be that the growth of a black middle-class should rather be interpreted has gaining access to a much wider array of financial services, and this should be interpreted as also gaining access to securities market.\(^7\)

\(^7\) We must also consider two other events. First, the advent of microloans that occurred in the mid 1990s should help consumers smooth consumption and acquire durable. The empirical evidence, however, is that these loans were not used to purchase durable. For example, Hurwitz and Luis (2007) document that, amongst the urban working class, microloans are mostly used for funerals, to pay off debt, for family emergencies, for education, and for transport. Bank loans are used more broadly, including for expenses related to housing, transport, and acquiring durable. Second, although difficult to quantify exactly, the AIDS epidemic must have force some households to devote growing fractions of their income to health related expenditures, and this should impede consumption smoothing. The AIDS epidemic must also have greatly reduced future income for some consumers, which should reduce the ability of some households to acquire loans.
2.3 Pre and Post Reform Business Cycle Moments

Table 1 displays standard business cycle moments for both South Korea and South Africa before and after financial deregulations. The data for this detailed in the Data Appendix. For this exercise, dating the reform is somewhat difficult. For Korea, the reform that concerns us started in 1993 and progressed at least until 1996, and was followed by the Asian crisis. To avoid any confusion, we simply remove the period from 1993 to 1997 from the data. Thus, our pre reform data includes data up to and including 1992, while our post reform data includes data after and including 1998. For South Africa, we are interested in the dismantling of apartheid that occurred between 1990 and 1996, we consider the pre reform period to extend to 1990 and the post reform period to start in 1996. For a broader view, Appendix Figure 5 presents a 20-year rolling window of the relative volatility of consumption and its components for the entire sample.

The observed moments are computed on the cyclical components extracted by the HP filter using a smoothing parameter of 100 (see the Data Appendix). Interestingly, the post reform moments indicate that the volatility of output declined for Korea, but rose for South Africa. For both countries, the relative volatility of aggregate consumption increased but the relative volatility of investment did not. For Korea, the relative volatility of durable consumption and net exports rose. For South Africa, the relative volatility of durable consumption and net exports declined.

3 The Closed Economy Model

3.1 The Closed Economy

The economy is populated by two types of households, that we label $r$ and $u$. Both types consume market and home produced goods. They engage in housework and supply labor to the market sector. Type $r$ households are relatively more impatient and poorer than their $u$ counterparts, they only have restricted access to asset markets. Type $u$ households are more patient and wealthier, and they have unrestricted access to asset markets. We exogenously assign a fraction $(1 - \omega)$ of the population to be of type $r$ and a fraction $\omega$ to be of type $u$, and we normalize total population to one.

The households derive utility from the consumption of a composite good $c_i^t$, and disutility from working $l_i^t$, where $i = r$ or $u$:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_i^t, l_i^t)$$

We note that Bekaert and Harvey (2005) select 1992 as their reform date, while Park (1996) and Bhattacharya and Patnaik (2013) select 1996 as the capital account liberalization date.
where $0 < \beta_i < 1$ is the subjective discount factor for households of type $i$ and $u(c^i_t, l^i_t) = \ln[c^i_t - (\varsigma/(1+\nu))l^i_{t+1}]$. In line with the empirical results in Becker and Mulligan (1997), the poorer type-$r$ households are impatient relative to the wealthier type-$u$ households: $\beta_r < \beta_u$.

The composite consumption good is given by

$$c^i_t = [\mu^i_m c^i_{mt}^{\epsilon_i} + \mu^i_h c^i_{ht}^{\epsilon_i}][1-1/\epsilon_i] \quad (2)$$

where $c^i_{mt}$ and $c^i_{ht}$ are consumption of market and home produced goods at time $t$ by consumers of type $i$, $\epsilon_i \geq 0$ is the elasticity of substitution between the two types of consumption goods, and $\mu^i_m \geq 0$ and $\mu^i_h \geq 0$ are measures of the bias toward market and home goods. Hours worked $l^i_t$ are allocated as follows

$$l^i_t = l^i_{mt} + l^i_{ht} \quad (3)$$

where $l^i_{mt}$ and $l^i_{ht}$ are time devoted to market and home work.

The home sector produces output $y^i_{ht}$ using capital $k^i_{ht}$ and labor $l^i_{ht}$:

$$y^i_{ht} = z^i_{ht}(k^i_{ht})^{\alpha_h}(l^i_{ht})^{1-\alpha_h} \quad (4)$$

where $0 < \alpha_h < 1$ denotes the capital share in home production and $z^i_{ht}$ is total factor productivity (TFP). Home capital evolves as

$$k^i_{ht+1} = \Phi(x^i_{ht}/k^i_{ht}) k^i_{ht} + (1-\delta_h)k^i_{ht} \quad (5)$$

where $x^i_{ht}$ denotes investment made in home capital (or, similarly, purchases of household durable) at period $t$, $0 < \delta_h < 1$ is a depreciation rate, and $\Phi(x^i_{ht}/k^i_{ht}) = x^i_{ht}/k^i_{ht} - (\phi/2)(x^i_{ht}/k^i_{ht} - \delta_h)^2$ implies investment adjustment costs, controlled by $\phi \geq 0$.

Home TFP follows a stochastic process described by

$$\ln(z^i_{ht}/\bar{z}_h) = \rho_i \ln(z^i_{ht-1}/\bar{z}_h) + \epsilon^i_{ht} \quad (6)$$

where $\epsilon^i_{ht}$ is a mean zero random variable with variance $\sigma^2_h > 0$, $\bar{z}_h > 0$ is the steady state value of home TFP, and $0 < \rho_i < 1$ denotes the persistence of the deviations of home TFP from its steady state value.

Finally, we note that home sector output is nonstorable and used only for consumption:

$$c^i_{ht} = y^i_{ht} \quad (7)$$

Type-$r$ households supply labor to firms against the market wage $W_t$. These households have restricted access to asset markets. In particular, they do not have access to the equity market, but
they can buy and sell one-period debt to finance their purchases. We denote this debt by $b^r_t$ and its price by $q_{bt}$. These households also purchase both market consumption goods $c^r_{mt}$ and household durable goods $x^r_{ht}$. Their period budget constraint is

$$c^r_{mt} + x^r_{ht} + b^r_t = (1 - \tau)W_t l^r_{mt} + q_{bt} b^r_{t+1},$$  \hspace{1cm} (8)$$

where $\tau$ denotes a proportional commuting cost. This cost creates a wedge between the marginal product of labor and the marginal rate of substitution between leisure and consumption.

All borrowing by type-$r$ households uses collateralized one-period ahead debt contracts. Type-$r$ households’ access to credit markets is imperfect in the sense that lenders are unable to enforce loan repayment. Thus, loan amounts are limited and borrowers need to provide their own wealth as collateral. Here, the collateral is the stock of durable goods or home capital owned by type-$r$ households. In the case of default, the lender is able to seize a fraction of the outside value of the stock and will only lend the amount consistent with the borrower’s incentive-compatibility constraint, such that there is no credit default in equilibrium. Denote $q^r_{ht+1}$ as the date $t+1$ relative price of a unit of installed home capital in terms of market goods. Then the debt contract will specify a repayment of $b^r_{t+1}$ that satisfies the collateral constraint:

$$b^r_{t+1} \leq \theta E_t \left[ q^r_{ht+1} \right] k^r_{ht+1}$$  \hspace{1cm} (9)$$

where the loan-to-value parameter $\theta > 0$ can be viewed as a credit multiplier: the credit constraint is relaxed as $\theta$ increases. In equilibrium, the relative price of a unit of installed home capital in terms of market goods is

$$q^r_{ht} = \left[ \Phi' \left( x^r_{ht}/k^r_{ht} \right) \right]^{-1}. \hspace{1cm} (10)$$

Type-$u$ households also supply labor to the market sector, but can freely access asset markets. As a result, type-$u$ households can buy and sell one-period bonds and market capital. As a result, they earn income from labor and from capital markets, where we denote the rental rate on market capital $k^u_{mt}$ by $R^u_t$. These households use their income to purchase market consumption goods $c^u_{mt}$, household durable goods $x^u_{ht}$, and market capital goods $x^u_{mt}$. Their period budget constraint is

$$c^u_{mt} + x^u_{ht} + x^u_{mt} + b^u_t = W_t l^u_{mt} + R^u_t k^u_{mt} + q_{bt} b^u_{t+1}.$$  \hspace{1cm} (11)$$

The market sector capital stock evolves as

$$k^u_{mt+1} = \Psi \left( x^u_{mt}/k^u_{mt} \right) k^u_{mt} + (1 - \delta_m) k^u_{mt} \hspace{1cm} (12)$$
where \( 0 < \delta_m < 1 \) is the depreciation rate of capital stock in the market sector, and \( \Psi \left( \frac{x_{mt}^u}{k_{mt}^u} \right) = x_{mt}^u/k_{mt}^u - (\psi/2)(x_{mt}^u/k_{mt}^u - \delta_m)^2 \), where \( \psi \geq 0 \) controls the investment adjustment cost.

Market output is produced with capital \( K_{mt} \) and labor \( L_{mt} \):

\[
Y_{mt} = Z_{mt} K_{mt}^{\alpha_m} L_{mt}^{1-\alpha_m}
\]  

(13)

where \( K_{mt} = \omega k_{mt}^u, L_{mt} = \omega l_{mt}^u + (1 - \omega)l_{mt}^r \). Market sector TFP is denoted \( Z_{mt} \). It follows

\[
\ln(Z_{mt}/\bar{Z}_m) = \rho_m \ln(Z_{mt-1}/\bar{Z}_m) + \nu_t
\]  

(14)

where \( \nu_t \) is a mean zero random variable with variance \( \sigma_m^2 > 0 \), \( \bar{Z}_m > 0 \) is the steady state value of market TFP, and \( 0 < \rho_m < 1 \) denotes the persistence of the deviations of market TFP.

Finally, to close the model, we let

\[
B_t = \omega b_t^u + (1 - \omega)b_t^r = 0.
\]  

(15)

Aggregate consumption can be decomposed into nondurables and durables: \( C_t = C_{nt} + C_{dt} \). Nondurable consumption includes consumption of nondurables by both types of households, \( C_{nt} = \omega c_{mt}^u + (1 - \omega)c_{mt}^r + \tau W_t l_{mt}^r \). Similarly, durable consumption includes purchases of home capital by both types of households, \( C_{dt} = \omega x_{ht}^u + (1 - \omega)x_{ht}^r \). Aggregate investment is \( I_t = \omega x_{mt}^u \). Finally, aggregate output is \( Y_t = Y_{mt} \).

### 3.2 Some Properties of the Closed Economy Model

We formally study the effects of financial reforms in the following section. For now, we wish to highlight a few features of the model. The Technical Appendix presents a detailed derivation of all the necessary optimality conditions.

In our model, a domestic financial reform can take two forms. The first is akin to an equity market reform where type-\( r \) households are granted full access to asset markets and essentially become type-\( u \) households. In our model, this is simply a rise in the proportion of the population \( \omega \) that is of type-\( u \). Admittedly, such a change involves more than simply granting access to asset markets as it also makes some consumers more patient (it raises their subjective discount factor). Nevertheless, this reform is expected to bring the emerging economy closer to an industrialized economy and improve consumption smoothing.

The second form of reform is more akin to a relaxation of credit and interest control. It occurs when the borrowing constraint faced by type-\( r \) households is relaxed. This is accomplished by a rise in the loan-to-value parameter \( \theta \). An increase in \( \theta \) relaxes the constraint in that it raises the
extent by which lenders can recover from type-\(r\) households and thus raises the amount that lenders are willing to lend.

In models where all households are of type \(u\), households' optimal asset choices are driven by their desires to smooth consumption and to invest in physical capital, both market and home capital. In our model, however, type-\(r\) households' optimal debt choice is affected by their impatience and constrained by the collateral constraint. We expect that this extra motive will tame the consumption smoothing motive, such that consumption will be more volatile. As a result, a rise in \(\theta\) should reinforce this extra motive at the detriment of the standard consumption smoothing motive.

Perhaps more importantly, the impatience and the collateral constraint also affect the accumulation of home capital (durable goods). For the type-\(r\) households, the marginal benefit of accumulating home capital includes not only the standard returns but also the extra benefit of relaxing the collateral constraint. This extra benefit amplifies the responses of durable purchases to shocks. This is in line with the amplification discussed in Kiyotaki and Moore (1997), but instead applied to home production. The difference is that, unlike most of the research where the collateral constraint amplifies the effects of shocks on market output, the amplification here is attached to home capital. As a result, the constraint amplifies the effects of shocks on the accumulation of durables and on nonmarket output (and thus to consumption). As a result, the collateral constraint should alter the fluctuations of consumption relative to those of market output and market income. Finally, the marginal benefit of loosening the collateral constraint is increasing in the loan-to-value parameter \(\theta\). Thus, we expect that a rise in the loan-to-value parameter \(\theta\) will further magnify the fluctuations of durable purchases.

Overall, we see two effects. First, the ability of type-\(r\) households to smooth consumption is somewhat impeded by their impatience. Second, the collateral constraint is likely to amplify the reaction of durable expenditures by type-\(r\) households, and this will likely reverberate to other expenditures while not necessarily affect output. Importantly, the amplification should depend on the loan-to-value parameter \(\theta\).

4 Reforms and Consumption Smoothing in the Closed Economy

4.1 Numerical Solution and Calibration

The properties of the model are studied numerically. Standard linear approximation methods are inappropriate because our model has an occasionally binding collateral constraint. We implement
the piecewise linear perturbation approach discussed in Guerrieri and Iacoviello (2015). As the authors point out, the advantage of this method is that it can easily handle cases with several state variables (as we have in our model). For this implementation, the collateral constraint binds in the deterministic steady state. The Technical Appendix provides a detailed derivation of the deterministic steady state.

The numerical solution requires values for all parameters. The model has many similarities with the model that Gollin, Parente, and Rogerson (2004) used to study the importance of farm and home work in explaining international productivity differences. Because of these similarities, we adopt similar parameter values. Table 1 presents parameter values consistent with annual data.

The preference parameters are set as follows. The type-$r$ household is impatient relative to the type-$u$ household: $\beta_r = 0.93$ and $\beta_u = 0.94$. This generates a steady state interest rate of roughly 6.4 percent. For both types, the elasticity of substitution between market and home produced goods is set at $\epsilon = 1.67$ as in Gollin, Parente, and Rogerson (2004). We set the preference share parameters for as follows. First, we define $\mu_i$, for $i = r$ and $u$, such that $\mu_i^{1/\epsilon} = \mu_i$ and $\mu_r^{1/\epsilon} = 1 - \mu_r$. We then adopt the strategy in Gollin, Parente, and Rogerson (2004) and set $\mu_r = 0.42$ and $\mu_u = 0.56$ to ensure that type-$r$ households put a higher weight on nonmarket activities and help deliver reasonable hours devoted to home work. We also set $\nu = 0.6$ such that the elasticity of labor supply is $1/\nu = 1.67$.

The values of the home and market production parameters also follow those of Gollin, Parente, and Rogerson (2004), but the exact values differ slightly. For home production, the only type specific parameter is the level of productivity $\bar{z}_h^r$. We set $\bar{z}_h^r = 1.05$ and $\bar{z}_h^u = 0.94$. These values ensure that type-$r$ households work more at home than type-$u$ households. To be specific, with these values, type-$r$ households devote 26 percent of their time to home work while type-$u$ households spend 24 percent of their time to home work. The capital share in the home sector $\alpha_h$ allows us to match the importance of durable purchases in consumption. For now, we set the capital share in the household sector to $\alpha_h = 0.20$ such that the share of durable is 9.00 percent, but we will explore the robustness of our results to this parameter. We set the depreciation rate to $\delta_h = 0.06$, as in Gollin, Parente, and Rogerson (2004). For market production, the capital share is set to $\alpha_m = 0.33$, the level of productivity to $\bar{Z}_m = 1$, and the depreciation rate to $\delta_m = 0.06$, as in Gollin, Parente, and Rogerson (2004). Finally, we set the adjustment cost parameters to low values $\phi = 0.5$ to help control the relative volatility of durable consumption expenditures. This also allows the price of the home capital, $q_{ht}^r$, to vary over time. For simplicity, we also set $\psi = 0.5$ but note that lower
values do not change aggregate investment volatility much.

The model has three other parameters that are absent in Gollin, Parente, and Rogerson (2004). We set the preference parameter associated to labor to $\zeta = 0.75$. We also set the commuting cost parameter to $\tau = 0.10$. This setting ensures that type-\(r\) household’s return to market work is lower than that of the type-\(u\) household. With this, the type-\(r\) household devotes about 24 percent of its time to market work, while the type-\(u\) household devotes about 29 percent of its time endowment to market work.

With these parameters in mind, we then set the following parameters to match the persistence and volatility of output and durable consumption expenditures in Korea over the entire sample. For this, we set the persistence parameters to $\rho_r = \rho_u = \rho_m = 0.67$ to match the persistence of output in South Korea. We also set the standard deviations to $\sigma_r = \sigma_u = 0.05$ and $\sigma_m = 0.02$ to match both the volatility of output and of consumption expenditures on durable goods.

Lastly, the two remaining parameters are the share of type-\(u\) households $\omega$ and the collateral constraint $\theta$. These parameters relate to financial reforms, and are thus studied more thoroughly. As we will see later, the model produces a typical business cycle for a variety of values of $\theta$ and $\omega$.

### 4.2 Domestic Financial Reforms in the Closed Economy

We simulate two different reforms. One reform, related to equity market access, requires that we raise the share $\omega$ of the population that is of type $u$, as type-\(u\) households have access to the equity market. Another reform, related to credit and interest control, requires that we raise the loan-to-value parameter $\theta$ so as to relax the collateral constraint on type-\(r\) households. Figure 2 plots the relative volatility of the cyclical fluctuations of consumption for different values of both $\omega$ and $\theta$. To be precise, panel (a) plots the standard deviation of the cyclical fluctuations in aggregate consumption relative to the standard deviation of the cyclical fluctuations of output, where the cyclical fluctuations correspond to the cyclical component extracted using the HP filter. As expected, the figure shows that a rise in $\omega$ for every $\theta$ unambiguously reduces the relative volatility of consumption. That is, financial reforms of this type ensure a larger extent of consumption smoothing and thus reduces consumption volatility. The figure also shows that changes in $\theta$ do not have a monotonic effect. At high values of $\omega$, changes in $\theta$ have little effect on aggregate consumption volatility simply because the fraction of households affected is too small. For lower values of $\omega$, a relaxation of the collateral constraint, a rise in $\theta$, may lower or raise consumption volatility. When $\theta$ is small, a rise in $\theta$ raises consumption volatility. Consumption volatility eventually reaches a maximum as $\theta$ rises. Once passed this maximum, further increases in $\theta$ reduce consumption volatility. Thus,
financial reforms of this type may ameliorate or deteriorate consumption smoothing, but only when a sizeable fractions of households (low \( \omega \)) are heavily constrained (low \( \theta \)).

As for panel (a), panels (b) and (c) display a decomposition of the relative volatility of consumption into its nondurable and durable components. The effects of relaxing the collateral constraint only occur for low values of \( \omega \) (high fraction of type-\( r \) households). Panel (b) shows that a relaxation of the collateral constraint raises the volatility of nondurable consumption, no matter the initial level of \( \theta \). Panel (c) shows that a relaxation of the collateral constraint produces a hump shaped response of the relative volatility of durable consumption expenditures, and is thus responsible for the hump shape witnessed in the relative volatility of aggregate consumption. For low values of \( \theta \), a relaxation of the collateral constraint magnifies substantially the volatility of durable consumption expenditures. For high values of \( \theta \), a further relaxation greatly reduces the volatility of durable consumption expenditures.

To better understand the effects of the collateral constraint, Figure 3 plots the relative volatility of consumption for the two types of households. For this experiment, we set \( \omega = 0.30 \). Panel (a) shows the relative volatility of consumption for type-\( r \) households, while panel (b) does so for type-\( u \) households. For type-\( r \) households, total consumption is \( C_t^r = C_{nt}^r + C_{dt}^r \) where nondurable consumption expenditure is \( C_{nt}^r = (1 - \omega)(c_{nt}^r + \tau W_t r_{nt}) \) and durable expenditure is \( C_{dt}^r = (1 - \omega)x_{ht}^r \), while income is \( Y_t^r = (1 - \omega)(1 - \tau)W_t r_{mt} \). For type-\( u \) households, total consumption is \( C_{nt}^u = C_{nt}^u + C_{dt}^u \) where nondurable consumption expenditure is \( C_{nt}^u = \omega c_{nt}^u \) and durable expenditure is \( C_{dt}^u = \omega x_{ht}^u \), while income is \( Y_t^u = \omega (W_t r_{nt} + R_t k_{mt}) \). The relative volatilities displayed in panel (a) confirm that the rise in the relative volatility of consumption observed in Figure 2 results from the behavior of consumption volatility for the type-\( r \) households. For these households, the relative volatility of nondurable consumption rises monotonically while that of durable consumption rises, reaches a peak, and then declines. This non-monotonicity is then transferred to the relative volatility of consumption. Panel (b) shows that the unconstrained type-\( u \) households smooth consumption better as type-\( r \) households become less constrained. That is, the relative volatility of total consumption declines as \( \theta \) rises (even though the relative volatility of durable displays a non-monotonicity similar to that of type-\( r \) households).

The pattern of the volatility of consumption for type-\( r \) households is consistent with our earlier discussion on the possible effects of financial reform. Starting from a low value of \( \theta \), a rise in \( \theta \) amplifies the effects of home productivity shocks on durable purchases. As a result, the volatility of durable purchases is magnified by a rise in \( \theta \).
4.3 The Quantitative Importance of Domestic Financial Reforms

Our results suggest that domestic financial reforms that loosen the collateral constraint for type-$r$ households can generate a rise in the relative volatility of consumption. In particular, the volatility of consumption mimics the volatility of durable expenditures for the constrained type-$r$ households. Here, we wish to analyze the quantitative importance of the mechanism that drives these results. We divide this discussion into two parts. First, we discuss the quantitative importance of the collateral constraint, and focus on the importance of the loan-to-value parameter $\theta$ in producing the hump shape response of the relative volatility of consumption. Then, we discuss the quantitative importance of home production in affecting the response of the relative consumption volatility to changes in $\theta$.

In the literature, the mechanism by which the collateral constraint affects the economy depends on the so-called amplification effect of the collateral constraint. We note here that this literature emphasizes the amplification of productivity shocks on output not on consumption. In the literature, the collateral constraint makes the accumulation of the collateral asset (physical market capital) and output (that relies on the collateral asset) more sensitive to productivity shocks. Several authors, however, have questioned the quantitative importance of the amplification effect. For example, Kocherlakota (2000) and Cordoba and Ripoll (2004) have suggested that the amplification effect is quantitatively small in models with concave preferences and concave production technologies. In contrast Mendicino (2012) who shows that the quantitative importance of the amplification effect depends crucially on the loan-to-value parameter $\theta$, a parameter that is ignored by Kocherlakota (2000) and Cordoba and Ripoll (2004). She documents that, in the economy considered by Cordoba and Ripoll (2004), the quantitative importance of the amplification effect is a hump shaped function of the loan-to-value parameter. More precisely, the persistence of the responses of output and investment to a productivity shock is a hump shaped function of the loan-to-value parameter. We note that in this economy, the impact effect of shocks depends mechanically on the productivity shock as consumers supply labor inelastically. In contrast, the persistence of the responses depend on the internal propagation mechanism embedded in the model by the collateral constraint. Importantly, the amplification effect is particularly small when the loan-to-value parameter is unity, the only value considered in Cordoba and Ripoll (2004).

To understand the underlying mechanism, Cordoba and Ripoll (2004) demonstrate that the importance of the collateral constraint depends on four factors: the steady state productivity gap between constrained and unconstrained households; the steady state share of the collateral in the
aggregate production function; the steady state fraction of output produced by constrained households; and the redistribution of collateral from unconstrained to constrained households originated by the shock. Mendicino (2012) argues that the loan-to-value parameter affects both the productivity gap and the fraction of output produced by constrained households. In that model, the steady state productivity gap is a decreasing function of the loan-to-value parameter. More precisely, the productivity of the collateral for constrained households is much larger than that of unconstrained agents for low values of the loan-to-value parameter. This gap however monotonically vanishes as the parameter rises to unity. All else equal, this makes the amplification effect quantitatively large at low values of the parameter, but much smaller at high values of the parameter. The steady state fraction of output produced by constrained households is an increasing function of the loan-to-value parameter. This makes the amplification effect quantitatively important at large values of the parameter but not at low values. As Mendicino (2012) shows, the combination of these effects gives rise to the hump shape.

As we have stated before, the literature focuses on the amplification of shocks on output, while we focus on the amplification of shocks on consumption relative to output. The distinction is important as larger fluctuations of output are likely to generate larger fluctuations of consumption, but not necessarily an increase in the volatility of consumption relative to that of output.

For this, the collateral constraint must have a larger effect on consumption than on output. In our model, the collateral asset is home capital (durable goods) and the output affected is home production. That said, as in Mendicino (2012), the effects of a rise of the loan-to-value parameter $\theta$ on the steady state productivity gap and the steady state fraction of income produced by constrained households generate a hump shape response in the quantitative importance of the collateral constraint.

First, the relevant productivity gap relates to the productivity of home capital, the collateral asset in our model. As our model includes labor, the productivity of home capital depends inversely on the relevant capital-labor ratio. For example, the marginal product of home capital is

$$R^i_h = \alpha_h z^i_h \left( k^i_h/l^i_h \right)^{1-\alpha_h},$$

where the deterministic steady state capital-labor ratios in the home sector for both households are

$$\frac{k^r_h}{l^r_h} = \left( \frac{\alpha_h}{1-\alpha_h} \right) \left[ \frac{\beta_r(1-\tau)\bar{W}}{1 - \beta_r(1-\delta_h) - \theta(\beta_u - \beta_r)} \right],$$

$$\frac{k^u_h}{l^u_h} = \left( \frac{\alpha_h}{1-\alpha_h} \right) \left[ \frac{\beta_u \bar{W}}{1 - \beta_u(1-\delta_h)} \right],$$

where the market wage is given by $\bar{W} = (1 - \alpha_m)Z_m \left[ \alpha_m Z_m / \bar{R}^k \right]^{\alpha_m/(1-\alpha_m)}$ for a market marginal
product of capital given by $\bar{R}_k = [1 - \beta_u(1 - \delta_m)] / \beta_u$.

We note that, for $\theta = 0$, the capital-labor ratio of the type-$r$ households is smaller than that of the type-$u$ households because $\beta_r < \beta_u$ (and $\tau \geq 0$). Accordingly, the marginal product of home capital for type-$r$ households is larger than that of type-$u$ households. Also, the capital-labor ratio for the type-$r$ households is an increasing function of the loan-to-value parameter $\theta$. Thus, the type-$r$ marginal product of home capital is a decreasing function of the loan-to-value parameter $\theta$, while the type-$u$ marginal product of home capital is invariant to $\theta$. Overall, a rise in the loan-to-value parameter $\theta$ reduces the productivity gap which lowers the quantitative importance of the collateral constraint.

Second, the relevant steady state fraction of output produced by constrained households corresponds to the steady state fraction of income accruing to type-$r$ households. This is the income that these households can use to accumulate the home capital, the asset subject to the collateral constraint.

Thus, we consider the effects of the loan-to-value parameter $\theta$ on the steady state fraction of market income accruing to the type-$r$ household relative to that accruing to type-$u$ household. In our model, the steady state market income for type-$r$ and type-$u$ households are $Y^r = (1 - \omega)(1 - \tau)W_{m}^r$ and $Y^u = \omega (W_{m}^u + R^k k_{m}^u)$, while total output is $Y_m = Z_m K_{m}^\alpha L_{m}^{1-\alpha} = W ((1 - \omega)l_m^r + \omega l_m^u) + R^k \omega k_{m}^u$. Figure 4 plots these steady state quantities for different values of $\theta$. Panel (a) documents that a rise in $\theta$ raises total output, while panel (b) shows that a rise in $\theta$ raises the income of type-$r$ households relative to that of type-$u$ households. Thus, a rise in $\theta$ raises the steady state level and fraction of income that accrues to type-$r$ households which raises the quantitative importance of the collateral constraint.

Overall, then, the trade-off between productivity gap and income share generated by a rise in $\theta$ explains the hump shape response of the relative volatility of durable expenditures for constrained type-$r$ households.\footnote{A similar effects arise when evaluating the effects of the commuting costs $\tau$ on the relative volatility of durable consumption expenditures. The Technical Appendix explores these effects.}

In addition to this, the rise in $\theta$ affects the relative volatility of nondurable consumption for the type-$r$ households as well as the relative volatilities of both durable expenditures and nondurable consumption for type-$u$ households. For both households, these effects arise in part from general equilibrium effects that alter the intertemporal prices of home and market capital (as well as wages). For type-$r$ households who are more directly affected, the response of the relative volatility of nondurable consumption to the rise in $\theta$ results from a variety of factors, including more volatile

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durable purchases that adds more volatility in the budget constraint, nonseparabilities in preferences that hides true smoothing, and the limited smoothing ability due to the collateral constraint and impatience. We note however that, as $\theta$ rises, the collateral constraint loosens such that type-$r$ consumer’s borrowing is not driven entirely to accumulate durable and can then be used to also smooth consumption. To see this, Figure 5 shows the fractions of periods that the collateral constraint binds. For low values of $\theta$, the constraint binds almost always. For high values of $\theta$, the constraint only binds half of the time. For type-$u$ households, the relative volatilities respond to general equilibrium effects.

The response of the relative consumption volatility to changes in the loan-to-value parameter $\theta$ is sensitive to the importance of the asset used as a collateral, which itself depends on the importance of home production. That is, our results depend on the importance of the stock of consumer durable held by type-$r$ consumers. In turn, purchases of consumer durable depend on key parameters of home production: the steady state TFP level of home production for the type-$r$ households, $\bar{z}_r^h$, and the capital share for home production, $\alpha_h$. To see this, Figure 6 displays the importance durable expenditures and the relative volatility of consumption for different values of $\bar{z}_r^h$ and $\alpha_h$, as well as different values of $\theta$. For this exercise, we set $\omega = 0.30$. Panels (a) and (b) present the steady state share of consumption expenditures devoted to durable goods. Unsurprisingly, increasing either the steady state level of TFP in the home production sector for type-$r$ households or the capital share in home production raises the deterministic steady state share of consumption expenditures devoted to durable. Panels (c) and (d) show the relative volatility of consumption. Raising either $\bar{z}_r^h$ or $\alpha_h$ raises the relative volatility of consumption for a given value of $\theta$. Also, higher values of either $\bar{z}_r^h$ or $\alpha_h$ are associated with a larger response of the relative volatility of consumption to changes in the loan-to-value parameter $\theta$.

5 Reforms in a Small Open Economy

We extend our analysis to a small open economy because the countries in Figure 1 are small open economies. In particular, our case studies below employ data from Korea and South Africa, both of which are now emerging economies that experience fluctuations in their current account and can thus vary their international borrowing. This latter ability might help smooth consumption and thus mitigate the effects of domestic financial liberalization on the relative volatility of consumption. In addition, emerging economies experience a higher propensity to crisis following international liberalizations. This should harm consumption smoothing and somewhat complement the effects
of domestic reforms on consumption smoothing. It would then appear important to verify the robustness and importance of our results to international liberalization. This would also allow us a better characterization of domestic financial liberalization in these economies.

5.1 A Small Open Economy

We extend our closed economy model to international borrowing and lending, and make it a small open economy model in the spirit of Neumeyer and Perri (2005). To do so, we replace the market clearing condition (15) by the interest rate equation

\[ R_{bt} = R^* + \chi \left[ \exp(B_{t+1} - B^*) - 1 \right] - \pi E_t \left[ \ln(Z_{mt+1}/\bar{Z}_m) \right], \]

(16)

where \( R^* \) is a constant international rate applied to the small open economy. In this interest rate equation, the latter two terms represent a country premium \( P_t = R_{bt} - R^* = \chi \left[ \exp(B_{t+1} - B^*) - 1 \right] - \pi E_t \left[ \ln(Z_{mt+1}/\bar{Z}_m) \right]. \) All else equal, the first term of this premium causes international lenders to raise the country premium when the country’s debt rises. This device also ensures that the model is stationary, as in Schmitt-Grohe and Uribe (2003) and Aguiar and Gopinath (2007). It can also be used to ensure a realistic persistence for the current account to output ratio (or the trade balance to output ratio), as argued in Garcia-Cicco, Pancrazi, and Uribe (2010). The second term of the premium causes international lenders to lower the country premium when the country’s ability to repay is improved, as in Neumeyer and Perri (2005) and Chang and Fernandez (2015). This helps deliver countercyclical interest rates, as documented in Uribe and Yue (2006). This also generates high consumption volatility. To see this, note that a persistent rise in productivity generates higher expected future productivity, a lower premium, and a lower interest rate. The higher productivity and the lower interest rate both stimulate consumption, so that consumption may be more volatile than income.

We calibrate this extended version as follows. First, we retain our previous calibration and simply set \( R^* = 1/\beta_u \) so that the deterministic steady state level of interest rate of the small open economy coincides with that of the closed economy. We then set \( B^* \) to yield a debt to output ratio of roughly 24 percent, similar to Korea’s average debt level in recent times. We parametrize the remaining two parameters in the spirit of Neumeyer and Perri (2005). For this, we select a low value of \( \chi = 0.01 \) and a value of \( \pi = 0.3 \) to obtain a volatility of consumption above unity. For both, controlling the volatility of durable purchases and market investment require higher values of the adjustment costs parameter. Accordingly, we set the adjustment costs parameter of the home sector to \( \phi = 2.0 \) and that of the market sector to \( \psi = 2.2. \)
The standard business cycle moments predicted by the open economy version of our model appear in Table 2. For this, we define the net export to output ratio to be $NX_t/Y_t = (Y_t - C_t - I_t)/Y_t$. The business cycle moments are constructed as before, except for the net export to output ratio that is not filtered. This calibration delivers reasonable business cycle moments. Importantly, consumption is more volatile than output. It also delivers a countercyclical net export to output ratio, as well as a reasonable persistence for the net export to output ratio. The latter result is interesting because Garcia-Cicco, Pancrazi, and Uribe (2010) argue that a high value of the sensitivity parameters $\chi$ is required to deliver a reasonable persistence. We note here that our model already embeds a mechanism to moderate international borrowing (via the collateral constraint) so that high levels of the sensitivity parameters $\chi$ are not required to deliver a reasonable persistence.

As for Figure 2, Figure 7 plots the relative volatility of consumption for different values of both $\omega$ and $\theta$. As before, a rise in $\omega$ for every $\theta$ unambiguously reduces the relative volatility of consumption, while a rise in $\theta$ for low values of $\omega$ raises the relative volatility of consumption. Panel (b) shows that a relaxation of the collateral constraint raises the volatility of nondurable consumption, while Panel (c) shows that a relaxation of the collateral constraint produces a hump shaped response of the relative volatility of durable consumption expenditures. We note that the effects on nondurable consumption is larger in this case such that the hump shape response of the relative volatility of durable expenditures does not extend to the relative volatility of aggregate consumption. As for Figure 3, Figure 8 displays the relative volatilities for both types of consumers and a value of $\omega = 0.30$. As before, then, the rise in the relative volatility of consumption observed in Figure 8 results from the behavior of consumption volatility for the type-$r$ household.

These results confirm that the effects of the collateral constraint on the relative volatility of consumption are robust to opening international borrowing. As a result, changes in the productivity $\bar{z}_h^r$ and the capital share in home production $\alpha_h$ affect the importance of the collateral constraint as before (these results can be obtained from the authors). The open economy version however introduces the parameters $\chi$ and $\pi$ that describes the sensitivity of the country premium to the debt level and expected future productivity. The Technical Appendix studies an alternative parametrization for the parameter $\chi$ and documents that the mechanics are robust to changes in this parameter. In particular, we set the sensitivity parameter $\chi$ in the spirit of Garcia-Cicco, Pancrazi, and Uribe (2010) and Chang and Fernandez (2015). For this, we select a high value of $\chi = 1.0$ and a high value of $\pi = 1.2$. Results are reported in Appendix Table 1, as well as Appendix Figures 2 and 3.

Here, instead, we focus our attention to the sensitivity parameter $\pi$. In Neumeyer and Perri
(2005) and Chang and Fernandez (2015), the parameter $\pi$ controls a reduced form device that generates a countercyclical real interest rate, the latter can explain high consumption volatility. The microfoundations of such a device relies on the theoretical literature lead by Eaton and Gersowitz (1981) and Arellano (2008) who argue that sovereign default probabilities, and thus country risk premium, are high when expectations of productivity are low. The relevance to our discussion of financial reforms is that the empirical work of Kaminsky and Reinhart (1999) and Ranciere, Tornell, and Westermann (2006) show that countries experience a higher propensity to crisis following financial liberalizations. This suggest that a financial liberalization would generate a higher sensitivity $\pi$.

Figure 9 displays the relative volatility of consumption for different values of both $\theta$ and $\pi$, and a value of $\omega = 0.30$. Panel (a) shows the relative volatility of aggregate consumption, while panels (b) and (c) show the decomposition by durability. Clearly, a rise in $\pi$ raises the relative volatility of aggregate consumption and its components. The figure also shows that a joint increase in both $\theta$ and $\pi$ can have dramatic effects on the relative volatility of consumption. Appendix Figure 4 presents the same results for the alternative calibration of the sensitivity parameter $\chi$.

### 5.2 Business Cycle Moments

Finally, we verify whether the model can reproduce the effects of a financial reform on the business cycle moments generated by the model. For these, we calibrate the model to approximate the experiences in Korea and South Africa prior to their reform and then simulate a reform, where the reform periods are discussed in Section 2. Evidently, these experiments are meant to be suggestive, rather than exhaustive.

Table 3 present the calibrations of the model used in the computations. The calibration aims to match the pre reform business cycle moments, but remains as close as possible to the benchmark calibration. Importantly, we retain the capital share $\alpha_h$ of the baseline calibration, which implies that durable expenditures form roughly 9 percent of consumption. To judge this choice, Appendix Figure 6 presents the consumption shares over our sample period. Using the Systems of National Accounts (SNA) definitions, the share of durable in the early 1990s was roughly 5 percent in Korea and 7 percent in South Africa. We note however that the SNA definitions impute rental expenditures in service expenditures, while our model would not. Removing all services raises the share of durables to roughly 10 percent in Korea and 11 percent in South Africa. In addition, the SNA definitions puts new residential construction as part of investment (dwellings). Adding those to our definition of durables, but keeping services, also raises the share of durable expenditures to
roughly 17 percent in Korea and 11 percent in South Africa. In what follows, we select a midway point of 9 percent to be somewhat less conservative than the exact SNA definition, but more conservative than simply removing all services or adding all dwellings.

There are three important parameters: The population share $\omega$, the loan-to-value parameter $\theta$, and the sensitivity parameter $\pi$. For Korea, we set $\omega$ to 30 percent. This choice is motivated by the fact that, in the Korean data, the top 20 percent of the household distribution saw a reduction in household debt after reform while the bottom 60 percent saw a rise in household debt. We then simply split the middle for the second top quintile who saw no change. For South Africa, we set $\omega$ to 20 percent. This choice is motivated by the fact that the 1995 Census reported that Whites formed roughly 13 percent of the population, and enjoyed very little unemployment and high levels of education. Interestingly, Indians who formed 3 percent of the population also enjoyed little unemployment and high levels of education. In contrasts, Coloureds and Africans displayed high unemployment and low education. We then use a value slightly hire than the sum of those numbers, which likely overstates the size of the type-a population.

Then, the simulated reforms will be modeled as increases in either the value of the loan-to-value parameter $\theta$ or in the value of the sensitivity $\pi$. The initial values for these parameters are set to match the pre reform business cycle moments. For Korea, the pre-reform values are $\theta = 0.001$ and $\pi = 0.30$. Admittedly, the value of $\theta$ is particularly low but in line with our reading that households were not allowed to borrow even with adequate collateral. For South Africa, the pre-reform values are $\theta = 0.50$ and $\pi = 0.40$. The value of $\theta$ is particularly high but required to match the high relative volatility of consumption observed in pre reform South Africa.

Table 4 and 5 report the business cycle moments for these experiments for Korea and South Africa. As before, the moments for Korea and South Africa are computed on the cyclical components extracted by the HP filter using a smoothing parameter of 100 (see the Data Appendix).

For both countries, the first experiment simulates a purely domestic financial reform. For this, we retain the pre-reform calibration but raise only the loan-to-value parameter to $\theta = 0.90$ for both Korea and South Africa. The second experiment simulates a strictly international financial reform. For this, we raise only the sensitivity parameter $\pi$ to $\pi = 0.95$ for both countries.

For Korea, the domestic reform raises the relative volatility of consumption and its components. This change, however, underpredicts the rise in the relative volatility of durable expenditures. In other dimensions, the reform lowers the volatility of output but fails to substantially raise the relative volatility of net exports. The international reform also raises the relative volatility
of consumption and its components, but also underpredicts the rise in the relative volatility of durable expenditures. Unfortunately, the reform raises the volatility of output but raises the relative volatility of net exports much more than the domestic reform.

For South Africa, the domestic reform raises the relative volatility of nondurable consumption but reduces the relative volatility of durable expenditures. The latter occurs because the economy starts near the top of the humpshaped behavior of the relative volatility of durable expenditures discussed in the previous section. The model however grossly understates the relative volatility of net exports. The international reform raises the relative volatility of consumption. Unfortunately, it generates a large increase in the relative volatility of durable expenditures and investment. It also generate a much higher relative volatility of net exports.

Taken together, the simulation results suggests that changes in the loan-to-value parameters or the sensitivity parameter alone may not provide full explanations of the rise in the relative volatility of consumption. However, these results provide some evidence that relaxing the collateral constraint will raise the relative volatility of consumption. It is interesting that a reform that raises the loan-to-value parameter can explain both a rise in the relative volatility of nondurable consumption and a reduction in the relative volatility of durable expenditures (as observed in South Africa). At the same time, the model may also require a rise in sensitivity of the country premium to expected future productivity to help explain the higher volatility.

6 Conclusion

We study how financial reforms or financial liberalization affect the extent of consumption smoothing in a dynamic stochastic general equilibrium model. The empirical literature highlights that financial reforms and development improves consumption smoothing in industrialized economies, but not in emerging economies.

Our closed economy model emphasizes consumer credit and home production. In accord with the empirical literature, our model predicts that some reforms ameliorate consumption smoothing. Our model also predicts that reforms that relax interest and credit controls may deteriorate consumption smoothing (raise the volatility of consumption relative to that of output) in emerging economies where a large share of the population is credit constrained and where home production is particularly important. These results also hold in a small open economy variant of the model that adds access to international financial markets.

Our paper offers some suggestive evidence of the effects of our mechanism in the financial
deregulations that occurred in the early to mid 1990s in Korea and South Africa. These results however are only suggestive. In particular, the mechanism by itself cannot fully explain the behavior of the relative volatility of consumption. In addition, our open economy version must acknowledge that financial reforms (especially capital account liberalization) raise the probability of foreign exchange and current account crisis. The latter is important because these crisis alter the cyclical behavior of country premiums and may well play an important role in shaping the relative volatility of consumption.
7 Data Appendix

Our first data source is World Bank, which reports Gross Domestic Product (GDP), consumption, investment, and next exports on an annual basis. We use data on GDP (net of government expenditure) and private consumption to compute the baseline data moments in Figure 1. The samples cover 1960 to 2015. The small open developed and industrial economies are Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, Norway, and Sweden. The developing and emerging economies are Argentina, Brazil, Chile, Colombia, Hong Kong, Indonesia, Korea (Republic of), Malaysia, Mexico, Peru, Philippines, Thailand, Venezuela, and South Africa.

Our second data source is OECD.Stat, which OECD reports data on household consumption for its member countries. OECD presents the total consumption expenditure of households following the COICOP classification and by durability. We have crossed checked with the NIPA classification, and find that most items in the OECD’s Semi-durable class are included in NIPA’s Durable class. So, we use OECD item Non-durable and Services to measure nondurable consumption, and attribute Durable and Semi-durable to durable consumption. The sample cover 1970 to 2014 for Korea and 1961 to 2014 for South Africa.
8 References


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### Table 1
Case Studies

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<tr>
<th></th>
<th>South Korea</th>
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Note: The numbers are the averages of 100 series of 500 periods simulated using the piecewise linear perturbation method of Guerrieri and Iacoviello (2015). Each relevant series is then detrended with the HP filter with a smoothness parameter of 100, except for the net export ratio. For Korea, the pre reform sample extends from 1970 to 1993, while the post reform sample extends from 1998 to 2014. For South Africa, the pre reform sample extends from 1970 to 1990, while the post reform sample extends from 1996 to 2014.
Table 3  
Case Studies Parameter Values

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Case Study: Korea

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<td><em>Net Export Ratio</em></td>
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<td>0.82</td>
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Note: The numbers are the averages of 100 series of 500 periods simulated using the piecewise linear perturbation method of Guerrieri and Iacoviello (2015). Each relevant series is then detrended with the HP filter with a smoothness parameter of 100, except for the net export ratio. For observed moments from Korea, the pre reform sample extends from 1970 to 1993, while the post reform sample extends from 1998 to 2014. For the model moments, the pre reform simulations assume that $\omega = 0.30$, $\theta = 0.001$, and $\pi = 0.30$. The post reform simulations use this calibration with the following changes. We set $\theta = 0.9$ for Post $\theta$ and set $\pi = 0.95$ for Post $\pi$. 

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Table 5
Case Study: South Africa

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Figure 1
Consumption Smoothing in Small Industrialized and Emerging Economies

Note: The figure shows the average ratio of the standard deviation of the cyclical component of consumption and the standard deviation of the cyclical component of output. The standard deviations are computed using 20-year rolling windows. The figures plots the cross-sectional averages as well as a one-standard deviation cross-sectional band. The small industrialized economies include Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, Norway, and Sweden. The emerging economies are Argentina, Brazil, Chile, Colombia, Hong Kong, Indonesia, Korea (Republic of), Malaysia, Mexico, Peru, Philippines, Thailand, Venezuela, and South Africa.
Figure 2
Financial Reforms and Consumption Volatility

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint and $\omega$ that controls the share of unconstrained type-$u$ households in the population.
Figure 3
Financial Reforms and Consumption Volatility: Type-\( r \) vs Type-\( u \)

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of income for different values of \( \theta \) that controls the tightness of the collateral constraint.
Figure 4
Financial Reform and Income

Note: Panel (a) shows the deterministic steady state level of output for different values of $\theta$ that controls the tightness of the collateral constraint. Panel (b) shows the deterministic steady state ratio of the income of type-$r$ households to the income of type-$u$ households for different values of $\theta$ that controls the tightness of the collateral constraint.
Figure 5
Financial Reform and Binding Collateral Constraint

Note: The figure shows the fraction of periods with a binding collateral constraint for different values of $\theta$ that controls the tightness of the collateral constraint.
Figure 6
Financial Reform and Home Production

Note: The figure shows the share of consumption expenditures devoted to durable as well as the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint. Panels (a) and (b) show the durable share for different values of home productivity level $\bar{z}_r$ and of the home production share of capital $\alpha_h$. Panels (c) and (d) show the relative volatility for different values of home productivity level $\bar{z}_r$ and of the home production share of capital $\alpha_h$. 

(a) Durable share and home productivity

(b) Durable share and home capital share

(c) Consumption volatility and home productivity

(d) Consumption volatility and home capital share
Figure 7
Financial Reforms and Consumption Volatility in the Open Economy

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint and $\omega$ that controls the share of unconstrained type-$u$ households in the population.
Figure 8
Financial Reforms and Consumption Volatility in the Open Economy:
Type-\( r \) vs Type-\( u \)

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of income for different values of \( \theta \) that controls the tightness of the collateral constraint.
Figure 9
Crisis, Liberalization, and Consumption Volatility in the Open Economy

![Figure 9](image)

(a) Aggregate consumption

(b) Nondurable

(c) Durable

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint and $\pi$ that controls the sensitivity of the country premium to expected future productivity.
Financial Reforms and Consumption Smoothing:
Technical Appendix

Martin Boileau and Tianxiao Zheng

September 2018

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9 Technical Appendix: The Model

For simplicity, we rewrite the period utility to be \( u(c^r_t, l^r_t) = u(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht}) \).

9.1 The Type-\( r \) Household

The type-\( r \) household’s problem (population mass \( 1 - \omega \)):

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t_r u(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht})
\]

subject to

\[
c^r_{mt} + x^r_{ht} + b^r_t = (1 - \tau_t)W_t l^r_{mt} + q^r_{ht} b^r_{t+1}
\]

\[
c^r_{ht} = y^r_{ht}
\]

\[
y^r_{ht} = z^r_{ht} (k^r_{ht})^{\alpha_h} (l^r_{ht})^{1-\alpha_h}
\]

\[
k^r_{ht+1} = \Phi (x^r_{ht}/k^r_{ht}) k^r_t + (1 - \delta_h)k^r_t
\]

\[
b^r_{t+1} \leq \theta E_t \left[ q^r_{ht+1} k^r_{ht+1} \right]
\]

The dynamic programming problem:

\[
V(b^r, k^r_h; s) = \max u(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht}) + \beta_r E \left[ V(b^{r'}, k^{r'}_h; s') \right]
\]

\[
+ \lambda^r ((1 - \tau)W_t l^r_{mt} + q^r b^r - c^r_m - x^r_h - b^r)
\]

\[
+ \gamma^r (z^r_h (k^r_h)^{\alpha_h} (l^r_h)^{1-\alpha_h} - c^r_h)
\]

\[
+ \eta^r_h (\Phi (x^r_h/k^r_h) k^r_h + (1 - \delta_h)k^r_h - k^r_{r'})
\]

\[
+ \zeta^r (\theta E_t \left[ q^r_{ht+1} k^r_{ht+1} - b^r \right])
\]

The necessary optimality conditions are:

\[
\lambda^r_t = u_1(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht})
\]

\[
\gamma^r_t = u_2(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht})
\]

\[-u_3(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht}) = \lambda^r_t (1 - \tau_t)W_t\]

\[-u_4(c^r_{mt}, c^r_{ht}, l^r_{mt}, l^r_{ht}) = \gamma^r_t (1 - \alpha_h)\]

\[
\lambda^r_t q^r_{ht} - \zeta^r_t = \beta_r E_t \left[ \lambda^r_{t+1} \right]
\]

\[
\lambda^r_t = \eta^r_{ht} \Phi' \left( \frac{x^r_{ht}}{k^r_{ht}} \right)
\]

\[
\eta^r_{ht} - \zeta^r_t \theta E_t \left[ q^r_{ht+1} \right] = \beta_r E_t \left[ \gamma^r_{t+1} \alpha_h \frac{q^r_{ht+1}}{k^r_{ht+1}} + \eta^r_{ht+1} \left( \Phi \left( \frac{x^r_{ht+1}}{k^r_{ht+1}} \right) - \Phi' \left( \frac{x^r_{ht+1}}{k^r_{ht+1}} \right) \frac{x^r_{ht+1}}{k^r_{ht+1}} + 1 - \delta_h \right) \right]
\]

\[
\zeta^r_t \left( \theta E_t \left[ q^r_{ht+1} \right] k^r_{ht+1} - b^r_{ht+1} \right) = 0
\]

with \( \zeta^r_t \geq 0 \) and \( E_t \left[ q^r_{ht+1} \right] k^r_{ht+1} \geq b^r_{ht+1} \) for \( q^r_{ht} = 1/\Phi' (x^r_h/k^r_{ht}) \). The expression \( u_j(c_{mt}, c_{ht}, l_{mt}, l_{ht}) \) denotes the partial derivative of the function \( u(\cdot) \) with respect to its \( j \)th arguments.
9.2 The Type-\textit{u} Household

The type-\textit{u} household’s problem (population mass \( \omega \)):

\[
\max E_0 \sum_{t=0}^{\infty} \beta^u_t u(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht})
\]

subject to

\[
c^u_{mt} + x^u_{ht} + x^u_{mt} + b^u_t = W_t l^u_{mt} + R^u_t k^u_{mt} + q_t b^u_t + 1
\]

\[
c^u_{ht} = y^u_{ht}
\]

\[
y^u_{ht} = z^u_{ht} (k^u_{ht})^\alpha_h (l^u_{ht})^{1-\alpha_h}
\]

\[
k^u_{ht+1} = \Phi (x^u_{ht}/k^u_{ht}) k^u_{ht} + (1 - \delta_h) k^u_{ht}
\]

\[
k^u_{mt+1} = \Psi (x^u_{mt}/k^u_{mt}) k^u_{mt} + (1 - \delta_m) k^u_{mt}
\]

The dynamic programming problem:

\[
V(b^u, k^u_{ht}, k^u_{mt}; s) = \max u(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht}) + \beta_u E \left[ V(b^{u'}, k^{u'}, s') \right]
\]

\[
+ \lambda^u \left( W_t l^u_{mt} + R^u_t k^u_{mt} + q_t b^u_t - c^u_t - x^u_t - x^u_{mt} - b^u_t \right)
\]

\[
+ \gamma^u \left( z^u_{ht} (k^u_{ht})^\alpha_h (l^u_{ht})^{1-\alpha_h} - c^u_{ht} \right)
\]

\[
+ \eta^u_h \left( \Phi (x^u_{ht}/k^u_{ht}) k^u_{ht} + (1 - \delta_h) k^u_{ht} - k^u_{ht} \right)
\]

\[
+ \eta^u_m \left( \Psi (x^u_{mt}/k^u_{mt}) k^u_{mt} + (1 - \delta_m) k^u_{mt} - k^u_{mt} \right)
\]

The necessary optimality conditions are

\[
\lambda^u_t = u_1(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht})
\]

\[
\gamma^u_t = u_2(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht})
\]

\[
-u_3(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht}) = \lambda^u_t W_t
\]

\[
-u_4(c^u_{mt}, c^u_{ht}, l^u_{mt}, l^u_{ht}) = \gamma^u_t (1 - \alpha_h) y^u_{ht}/l^u_{ht}
\]

\[
\lambda^u_t = \lambda^u_{ht} \Phi'(x^u_{ht}/k^u_{ht})
\]

\[
\lambda^u_t = \eta^u_{mt} \Psi'(x^u_{mt}/k^u_{mt})
\]

\[
\lambda^u_t q_t = \beta_u E_t \left[ \lambda^u_{t+1} \right]
\]

\[
\eta^u_{ht} = \beta_u E_t \left[ \gamma^u_{t+1} \alpha_h y^u_{ht+1}/k^u_{ht+1} + \eta^u_{ht+1} \left( \Phi \left( y^u_{ht+1}/k^u_{ht+1} \right) - \Phi' \left( y^u_{ht+1}/k^u_{ht+1} \right) \frac{x^u_{ht+1}}{k^u_{ht+1}} \right) + 1 - \delta_h \right]
\]

\[
\eta^u_{mt} = \beta_u E_t \left[ \lambda^u_{t+1} R^u_{t+1} + \eta^u_{mt+1} \left( \Psi \left( x^u_{mt+1}/k^u_{mt+1} \right) - \Psi' \left( x^u_{mt+1}/k^u_{mt+1} \right) \frac{x^u_{mt+1}}{k^u_{mt+1}} \right) + 1 - \delta_m \right]
\]
9.3 Closing the Model

Production of market goods:
\[ Y_{mt} = Z_{mt}K_{mt}^{\alpha_m}L_{mt}^{1-\alpha_m} \]
where \( K_{mt} = \omega k_{mt}^u \) and \( L_{mt} = \omega l_{mt}^u + (1 - \omega)l_r^u \).

As a result
\[ W_t = (1 - \alpha_m)Y_{mt}/L_{mt} \]
\[ R_t^k = \alpha_m Y_{mt}/K_{mt} \]

For the bonds market, we have
\[ B_t = \omega b_t^u + (1 - \omega)b_r^u \]
and
\[ R_{bt} = 1/q_{bt} \]

We then have 2 possibilities:
- Closed Economy: \( B_t = 0 \)
- Small Open Economy: \( R_{bt} = R^* + \chi [\exp(B_{t+1} - B^*) - 1] - \pi E_t [\ln(Z_{mt+1}/\bar{Z}_m)] \)

Some derivatives:
\[ u(c_t, l_t) = \ln[c_t - (\varsigma/(1 + \nu))l_t^{1+\nu}] \]
\[ c_t = \left[ \mu_m^{-\frac{1}{\varepsilon}}c_{mt}^{-\frac{1}{\varepsilon}} + \mu_h^{\frac{1}{\varepsilon}}c_{ht}^{\frac{1}{\varepsilon}} + \frac{1}{\varepsilon - 1} \right]^{\varepsilon - 1} \]
\[ l_t = l_{mt} + l_{ht} \]

Then:
\[ u_{ct} = 1/[c_t - (\varsigma/(1 + \nu))l_t^{1+\nu}] \]
\[ u_{lt} = -u_{ct}\varsigma l_t^\nu \]
\[ u_1(c_{mt}, c_{ht}, l_{mt}, l_{ht}) = u_{ct}\mu_m^{1/\varepsilon} [c_t/c_{mt}]^{1/\varepsilon} \]
\[ u_2(c_{mt}, c_{ht}, l_{mt}, l_{ht}) = u_{ct}\mu_h^{1/\varepsilon} [c_t/c_{ht}]^{1/\varepsilon} \]
\[ -u_3(c_{mt}, c_{ht}, l_{mt}, l_{ht}) = u_{ct}\varsigma l_t^\nu \]
\[ -u_4(c_{mt}, c_{ht}, l_{mt}, l_{ht}) = u_{ct}\varsigma l_t^\nu \]

Also, from
\[ \Phi(x_{ht}/k_{ht}) = x_{ht}/k_{ht} - (\phi/2)(x_{ht}/k_{ht} - \delta_h)^2 \]
\[ \Psi(x_{mt}/k_{mt}) = x_{mt}/k_{mt} - (\psi/2)(x_{mt}/k_{mt} - \delta_m)^2 \]
we get
\[ \Phi'(x_{ht}/k_{ht}) = 1 - \phi(x_{ht}/k_{ht} - \delta_h) \]
\[ \Psi'(x_{mt}/k_{mt}) = 1 - \psi(x_{mt}/k_{mt} - \delta_m) \]
9.4 The Deterministic Steady State

The static equations for type-$r$ households are:

\[
\begin{align*}
\lambda^r &= u_1(c^r_m, c^r_h, l^r_m, l^r_h) \\
\gamma^r &= u_2(c^r_m, c^r_h, l^r_m, l^r_h) \\
- u_3(c^r_m, c^r_h, l^r_m, l^r_h) &= \lambda^r (1 - \tau) W \\
- u_4(c^r_m, c^r_h, l^r_m, l^r_h) &= \gamma^r W^r_h \\
\lambda^r &= \eta_h^r \Phi'(x^r_h/k^r_h) \\
c^r_h &= y^r_h \\
y^r_h &= z^r_h (k^r_h)^{\alpha_h} (l^r_h)^{1 - \alpha_h}
\end{align*}
\]

The static equations for type-$u$ households are:

\[
\begin{align*}
\lambda^u &= u_1(c^u_m, c^u_h, l^u_m, l^u_h) \\
\gamma^u &= u_2(c^u_m, c^u_h, l^u_m, l^u_h) \\
- u_3(c^u_m, c^u_h, l^u_m, l^u_h) &= \lambda^u W \\
- u_4(c^u_m, c^u_h, l^u_m, l^u_h) &= \gamma^u W^u_h \\
\lambda^u &= \eta^u_h \Phi'(x^u_h/k^u_h) \\
\lambda^u &= \eta^u_m \Psi'(x^u_m/k^u_m) \\
c^u_h &= y^u_h \\
y^u_h &= z^u_h (k^u_h)^{\alpha_h} (l^u_h)^{1 - \alpha_h}
\end{align*}
\]

The static equations for closing the model:

\[
\begin{align*}
Y_m &= Z_m K^\alpha_m L^{1-\alpha_m} \\
W &= (1 - \alpha_m) Y_m/L_m \\
R^k &= \alpha_m Y_m/K_m \\
q^r_h &= \left[ \Phi'(x^r_h/k^r_h) \right]^{-1}
\end{align*}
\]

Static definitions:

\[
\begin{align*}
K_m &= \omega k^u_m \\
L_m &= \omega l^u_m + (1 - \omega) l^r_m \\
B &= \omega b^u + (1 - \omega) b^r \\
R_b &= 1/q_b \\
W^r_h &= (1 - \alpha_h) y^r_h/l^r_h \\
R^r_h &= \alpha_h y^r_h/k^r_h \\
W^u_h &= (1 - \alpha_h) y^u_h/l^u_h \\
R^u_h &= \alpha_h y^u_h/k^u_h
\end{align*}
\]
The dynamic equations for type-\(r\) households are:

\[
\begin{align*}
\lambda^r q_b - \zeta^r &= \beta_r \lambda^r \\
\eta^r h - \zeta^r \theta q^r_b &= \beta_r \left[ \gamma^r R^r_h + \eta^r_{ht} (\Phi (x^r_h/k^r_h) - \Phi' (x^r_h/k^r_h) x^r_h/k^r_h + 1 - \delta_h) \right] \\
\zeta^r (\theta q^r_h k^r_h - b^r) &= 0 \\
c^r_m + x^r_h + b^r = (1 - \tau)Wl^r_m + q_b b^r \\
k^r_h = \Phi (x^r_h/k^r_h) k^r_h + (1 - \delta_h)k^r_h
\end{align*}
\]

The dynamic equations for type-\(u\) households are:

\[
\begin{align*}
\lambda^u q_b &= \beta_u \lambda^u \\
\eta^u h &= \beta_u \left[ \gamma^u R^u_h + \eta^u_{ht} (\Phi (x^u_h/k^u_h) - \Phi' (x^u_h/k^u_h) x^u_h/k^u_h + 1 - \delta_h) \right] \\
\eta^u m &= \beta_u \left[ \lambda^u R^u_k + \eta^u_{km} (\Psi (x^u_m/k^u_m) - \Psi' (x^u_m/k^u_m) x^u_m/k^u_m + 1 - \delta_m) \right] \\
c^u_m + x^u_h + x^u_m + b^u &= Wl^u_m + R^u_k k^u_m + q_b b^u \\
k^u_h &= \Phi (x^u_h/k^u_h) k^u_h + (1 - \delta_h)k^u_h \\
k^u_m &= \Psi (x^u_m/k^u_m) k^u_m + (1 - \delta_m)k^u_m
\end{align*}
\]

Closed Economy:

\[
B = 0
\]

Small Open Economy:

\[
R_b = R^* + \chi [\exp((B - B^*) - 1) - \pi \ln(\bar{Z}_m/\bar{Z}_m)]
\]

The deterministic steady state values will be represented as \(\bar{x}\), the deterministic steady state value of \(x\). The obtain the steady state:

1. From the following equations,

\[
\begin{align*}
\lambda^u q_b &= \beta_u \lambda^u \\
R^u_b &= 1/q_b
\end{align*}
\]

obtain

\[
\begin{align*}
\bar{q}_b &= \beta_u \\
\bar{R}_b &= 1/\beta_u
\end{align*}
\]

For the closed economy:

\[
\bar{B} = 0
\]

For the small open economy, we use

\[
R_b = R^* + \chi [\exp((B - B^*) - 1) - \pi \ln(\bar{Z}_m/\bar{Z}_m)]
\]

to obtain

\[
\bar{B} = B^* + \ln \left(1 + (\bar{R}_b - R^*)/\chi\right)
\]
2. We impose the following conditions,\( \Phi(\delta_h) = \delta_h, \Psi(\delta_m) = \delta_m, \) and \( \Phi'(\delta_h) = \Psi'(\delta_m) = 1, \) and rewrite the accumulation equations,

\[
\begin{align*}
k^r_h &= \Phi\left(\frac{x^r_h}{k^r_h}\right) k^r_h + (1 - \delta_h)k^r_h \\
k^u_h &= \Phi\left(\frac{x^u_h}{k^u_h}\right) k^u_h + (1 - \delta_h)k^u_h \\
k^u_m &= \Psi\left(\frac{x^u_m}{k^u_m}\right) k^u_m + (1 - \delta_m)k^u_m,
\end{align*}
\]

to obtain \( x^r_h = \delta_h k^r_h, \) \( x^u_h = \delta_h k^u_h \) and \( x^u_m = \delta_m k^u_m. \) Then, we use

\[
\begin{align*}
\lambda^r &= \eta^r_h \Phi'(\delta_h) \\
\lambda^u &= \eta^u_h \Phi'(\delta_h) \\
\lambda^m &= \eta^u_m \Psi'(\delta_m) \\
d^r_h &= \left[\Phi'(\delta_h)\right]^{-1}
\end{align*}
\]

to obtain \( \lambda^r = \eta^r_h, \lambda^u = \eta^u_h = \eta^u_m, \) and \( d^r_h = 1. \)

3. From the following

\[
\eta^u_m = \beta_u \left[ \lambda^u R_k + \eta^u_m \left( \Psi\left(\frac{x^u_m}{k^u_m}\right) - \Psi'(\frac{x^u_m}{k^u_m})x^u_m/k^u_m + 1 - \delta_m \right) \right]
\]

and previous results, we find that

\[
\bar{R}_k = \bar{R}_b - (1 - \delta_m) = \left[ 1 - \beta_u (1 - \delta_m) \right] / \beta_u
\]

Also, from

\[
\begin{align*}
Y_m &= Z_m K_m \alpha_m L_m^{1 - \alpha_m} \\
W &= (1 - \alpha_m) Y_m / L_m \\
R_k &= \alpha_m Y_m / K_m
\end{align*}
\]

we obtain

\[
\bar{W} = (1 - \alpha_m) Z_m \left[ \frac{\alpha_m Z_m}{R_k} \right]^{\alpha_m / (1 - \alpha_m)}
\]

Finally, we define the market capital-labor ratio to be

\[
\mathcal{H}_m \equiv K_m / L_m = \left( \frac{\alpha_m}{1 - \alpha_m} \right) \left[ \frac{\beta_u \bar{W}}{1 - \beta_u (1 - \delta_m)} \right]
\]

such that \( K_m = \mathcal{H}_m L_m. \)

4. Let \( u^i_3 = u^i_4 \) for \( i = r, \) \( u \) because \( l^i = l^i_h + l^i_m. \) From

\[
\begin{align*}
-u_3(c^u_m, c^u_h, l^u_m, l^u_h) &= \lambda^u W \\
-u_4(c^u_m, c^u_h, l^u_m, l^u_h) &= \gamma^u W^u_h
\end{align*}
\]

\[
\begin{align*}
\eta^u_h &= \beta_u \left[ \gamma^u R^u_h + \eta^u_h \left( \Phi\left(\frac{x^u_h}{k^u_h}\right) - \Phi'(\frac{x^u_h}{k^u_h})x^u_h/k^u_h + 1 - \delta_h \right) \right] \\
\eta^u_m &= \beta_u \left[ \lambda^u R_k + \eta^u_m \left( \Psi\left(\frac{x^u_m}{k^u_m}\right) - \Psi'(\frac{x^u_m}{k^u_m})x^u_m/k^u_m + 1 - \delta_m \right) \right]
\end{align*}
\]
and previous results, obtain

\[ R^u_h/W^u_h = [\bar{R}_k + (\delta_h - \delta_m)] / \bar{W} \]

Given the definitions:

\[ W^u_h = (1 - \alpha_h) y^u_h / l^u_h \]
\[ R^u_h = \alpha_h y^u_h / k^u_h \]
\[ y^u_h = z^u_h (k^u_h)^{\alpha_h} (l^u_h)^{1-\alpha_h} \]

then

\[ R^u_h/W^u_h = [\alpha_h/(1 - \alpha_h)] l^u_h / k^u_h \]

Using the above, we define the type-\( u \) household’s home capital-labor ratio to be

\[ \mathcal{H}^u_h \equiv k^u_h/l^u_h = \left( \frac{\alpha_h}{1 - \alpha_h} \right) \left[ \frac{\beta_u \bar{W}}{1 - \beta_u(1 - \delta_h)} \right] \]

such that \( k^u_h = \mathcal{H}^u_h l^u_h \). We then have

\[ \bar{R}^u_h = \alpha_h z^u_h (\mathcal{H}^u_h)^{-(1-\alpha_h)} \]
\[ \bar{W}^u_h = (1 - \alpha_h) z^u_h (\mathcal{H}^u_h)^{\alpha_h} \]

From the following:

\[ -u_3(c^r_m, c^r_h, l^r_m, l^r_h) = \lambda^r (1 - \tau) W \]
\[ -u_4(c^r_m, c^r_h, l^r_m, l^r_h) = \gamma^r W^r_h \]
\[ \lambda^r q^r - \zeta^r = \beta_r \lambda^r \]
\[ \eta^r_h - \zeta^r \theta q^r_h = \beta_r \left[ \gamma^r R^r_h + \eta^r_h (\Phi(x^r_h/k^r_h) - \Phi'(x^r_h/k^r_h)x^r_h/k^r_h + 1 - \delta_h) \right] \]

and previous results, we have

\[ \zeta^r = \lambda^r (\beta_u - \beta_r) > 0 \]

and

\[ R^r_h/W^r_h = [1 - \theta (\beta_u - \beta_r) - \beta_r(1 - \delta_h)] / [\beta_r(1 - \tau)\bar{W}] \]

Given the definitions:

\[ W^r_h = (1 - \alpha_h) y^r_h / l^r_h \]
\[ R^r_h = \alpha_h y^r_h / k^r_h \]
\[ y^r_h = z^r_h (k^r_h)^{\alpha_h} (l^r_h)^{1-\alpha_h} \]

then

\[ R^r_h/W^r_h = [\alpha_h/(1 - \alpha_h)] l^r_h / k^r_h \]
We define the type-$r$ household’s home capital-labor ratio to be

$$H_r^h \equiv k_r^h / l_r^h = \left( \frac{\alpha_h}{1 - \alpha_h} \right) \left[ \frac{\beta_r (1 - \tau) \bar{W}}{1 - \beta_r (1 - \delta_h) - \theta (\beta_u - \beta_r)} \right]$$

such that $k_r^h = H_r^h l_r^h$, and

$$\bar{R}_r^h = \alpha_h z_r^h (H_r^h)^{-(1 - \alpha_h)}$$
$$\bar{W}_r^h = (1 - \alpha_h) z_r^h (H_r^h)^{\alpha_h}$$

5. Here, we define some prices. Importantly, we define a price index $P$ such that $Pc^i = c_m^i + p_h^i c_h^i$, where $p_h^i$ is the relative price of home goods. Using $c^i = [\mu_m^i c_m^i + \mu_h^i \bar{z}_h^i (H_r^h)^{\alpha_h} c_m^i]^{1/\varepsilon}$, we find that the relative price is

$$p_h^i = \left( \frac{\mu_h^i}{\mu_m^i} \right)^{1/\varepsilon} \left[ \frac{c_m^i}{c_h^i} \right]^{1/\varepsilon}$$

This, as usual, leads to a price index

$$P^i = [\mu_m^i + \mu_h^i (p_h^i)^{1-\varepsilon}]^{1/(1-\varepsilon)}$$

and two demand functions

$$c_m^i = \mu_m^i (P^i)^{\varepsilon} c^i$$
$$c_h^i = \mu_h^i (P^i/p_h^i)^{\varepsilon} c^i$$

6. To progress, we need to make some assumptions about preferences:

$$u(c^i, l^i) = \ln[c^i - (\varsigma/(1 + \nu)) l^{1 + \nu}]$$

Then:

$$u_1^i = 1/[c^i - (\varsigma/(1 + \nu)) l^{1 + \nu}]$$
$$u_2^i = -u_3^i \varsigma l^{\nu}$$
$$u_3^i = u_4^i \varsigma l^{\nu}$$
$$u_4^i = u_5^i \varsigma l^{\nu}$$

We first note that

$$\frac{u_2^i}{u_1^i} = \gamma^i \lambda^i = \left( \frac{\mu_h^i}{\mu_m^i} \right)^{1/\varepsilon} \left[ \frac{c_m^i}{c_h^i} \right]^{1/\varepsilon} = p_h^i$$

Using our previous results:

$$\bar{p}_r^h = (1 - \tau) \bar{W} / \bar{W}_r^h$$
$$\bar{p}_h^u = \bar{W} / \bar{W}_h^u$$

and

$$\bar{P}^i = [\mu_m^i + \mu_h^i (\bar{p}_h^i)^{1-\varepsilon}]^{1/(1-\varepsilon)}$$
7. To find the time allocation, note that

\[-u_3(c_r^m, c_h^r, l_m^r, l_h^r) = \lambda^r (1 - \tau)W\]
\[-u_3(c_m^o, c_h^m, l_m^o, l_h^o) = \lambda^o W\]

and previous relations, we have

\[-u_3^i = \frac{\varsigma l^{\nu}}{u^{1/\varepsilon} [c^{i}/c_m^{i}]^{1/\varepsilon}}\]

It is easy to verify that \(\mu_m^{1/\varepsilon} [c^{i}/c_m^{i}]^{1/\varepsilon} = 1/P^i\). Thus:

\[\varsigma l^{\nu} P^r = (1 - \tau)W\]
\[\varsigma l^{\mu} P^u = W\]

or

\[\bar{l}^r = ((1 - \tau)/\varsigma)^{1/\nu} \left[\bar{W}/\bar{P}^r\right]^{1/\nu}\]
\[\bar{l}^u = (1/\varsigma)^{1/\nu} \left[\bar{W}/\bar{P}^u\right]^{1/\nu}\]

8. Using the budget constraint

\[c_r^o + x_r^o + b^r = (1 - \tau)Wl_m^r + q_b b^r\]

and our previous results. First, note that

\[\zeta^r = \lambda^r (\beta_u - \beta_r) > 0\]
\[\zeta^r (\theta q_b^r k^r_h - b^r) = 0\]

implies that \(b^r = \theta k^r_h\). We also know that \(x_r^r = \delta_h^r k^r_h\).

To progress, recall that that \(k^r_h = H^r l^r_h\). Thus,

\[b^r = \theta k^r_h = \theta H^r l^r_h\]
\[x_r^r = \delta_h^r k^r_h = \delta_h H^r l^r_h\]
\[c_r^r = \left(\frac{\mu_m^r}{\mu_h^r}\right) (p_h^r)^{e_r} c_r^r\]
\[c_h^r = z_h^r (k^r_h / l^r_h)^{\alpha_h} l^r_h = z^r (H^r)^{\alpha_h} l^r_h\]
\[l^r_m = l^r - l^r_h\]

In the budget:

\[\left[ \left(\frac{\mu_m^r}{\mu_h^r}\right) (p_h^r)^{e_r} z^r (H^r)^{\alpha_h} \right] l^r_h + (\delta_h H^r) l^r_h + [(1 - \bar{q}_b) (\theta H^r)] l^r_h = (1 - \tau)\bar{W}(l^r - l^r_h)\]

or

\[\bar{l}_h = \frac{(1 - \tau)\bar{W}l^r}{\left[ \left(\frac{\mu_m^r}{\mu_h^r}\right) (p_h^r)^{e_r} z^r (H^r)^{\alpha_h} \right] + [(\delta_h + (1 - \bar{q}_b)\theta) H^r] + (1 - \tau)\bar{W}}\]
From this, we get:

\[
\begin{align*}
\bar{r}_m &= \bar{r} - \bar{r}_h \\
\bar{k}_h &= \mathcal{H}_h \bar{r}_h \\
y_h^r &= z_h^u (k_h^u)_{\alpha h} (\bar{r}_h)^{1-\alpha_h} \\
\bar{c}_h &= \bar{y}_h \\
x_h^r &= \delta_h \bar{k}_h^r \\
\bar{b}_h &= \theta_k \bar{k}_h^r \\
\bar{c}_m^r &= \left( \frac{\mu_m^u}{\mu_h^u} \right) (\bar{p}_h^r)^{\zeta} \bar{c}_h \\
\bar{c}^r &= \left[ \mu_m^u \right] (c_h^u)^{\frac{\zeta - 1}{\zeta}} + \mu_h^u \left[ (c_h^u)^{\frac{\zeta - 1}{\zeta}} \right]^{\frac{\zeta - 1}{\zeta}} \\
\bar{\lambda}^r &= (\bar{P}^r)^{-1} / [\bar{c}^r - (\zeta/(1 + \nu))(\bar{r})^{1+\nu}] \\
\bar{\gamma}^r &= \bar{p}_h^u \bar{\lambda}^r \\
\bar{\eta}^r &= \bar{\lambda}^r \\
\bar{\zeta}^r &= \bar{\lambda}^r (\beta_u - \beta_r)
\end{align*}
\]

9. Using the budget constraint:

\[
c_m^u + x_h^u + x_m^u + b^u = W l_m + R_k k_m + q_b b^u
\]

First, note that, from the definition of \( B = \omega b^u + (1 - \omega) b^r \), we have

\[
\bar{b}^u = (\bar{B} - (1 - \omega)\bar{b}^r) / \omega
\]

such that

\[
c_m^u + x_h^u = \bar{W} \bar{l}^u - \bar{W} l_h^u + (\bar{R}_k - \delta_m) k_m^u - (1 - \bar{q}_b) \bar{b}^u
\]

Then:

\[
\begin{align*}
c_h^u &= z_h^u (k_h^u/l_h^u)^{\alpha h} l_h^u = z_h^u (\mathcal{H}_h^u)^{\alpha h} l_h^u \\
c_m^u &= \left( \frac{\mu_m^u}{\mu_h^u} \right) (\bar{p}_h^u)^{\zeta} \bar{c}_h \\
x_h^u &= \delta_h \mathcal{H}_h^u l_h^u \\
l_m^u &= l^u - l_h^u \\
L_m &= \omega l_m^u + (1 - \omega) \bar{l}_m^r \\
k_m^u &= [1/\omega] \mathcal{H}_m L_m = \mathcal{H}_m l^u + ((1 - \omega)/\omega) \mathcal{H}_m \bar{l}_m^r - \mathcal{H}_m \bar{l}_m^u
\end{align*}
\]

To simplify, define

\[
\Omega_1^u \equiv \left[ \left( \frac{\mu_m^u}{\mu_h^u} \right) (\bar{p}_h^u)^{\zeta} z_h^u (\mathcal{H}_h^u)^{\alpha h} \right] + (\delta_h \mathcal{H}_h^u)
\]

10
In the budget:

\[ \Omega^m_{ht} = Wl^m_t - Wl^m_h + (R_k - \delta_m) \left[ \mathcal{H}_m l^m_{ht} + ((1 - \omega)/\omega) \mathcal{H}_m l^m_{ht} - \mathcal{H}_m l^m_{ht} \right] - (1 - \bar{q}_b) \bar{b}^u \]

such that

\[ \bar{l}^u_{ht} = \frac{Wl^m_t + (R_k - \delta_m) \left[ \mathcal{H}_m l^m_{ht} + ((1 - \omega)/\omega) \mathcal{H}_m l^m_{ht} - \mathcal{H}_m l^m_{ht} \right] - (1 - \bar{q}_b) \bar{b}^u}{\left( \frac{\mu_h^m}{\mu_h^u} \right) \left( \bar{p}_{ht}^u \right)^{\varepsilon_h} - \bar{y}_{ht} \mathcal{H}_h^u \alpha_h} + \left( \delta_h \mathcal{H}_h^u \right) + W + (R_k - \delta_m) \mathcal{H}_m \]

From these, we get:

\[ \bar{r}_{ht} = \bar{l}^u_{ht} - \bar{r}_{ht} \]
\[ \bar{k}_{ht} = \mathcal{H}_h^u \bar{r}_{ht} \]
\[ \bar{y}_{ht} = Z_h^u \left( \bar{k}_{ht} \right)^{\alpha_h} \left( \bar{l}^u_{ht} \right)^{1-\alpha_h} \]
\[ \bar{c}_{ht}^u = \bar{y}_{ht} \]
\[ \bar{x}_{ht} = \delta_h \bar{k}_{ht} \]
\[ \bar{L}_m = \omega \bar{l}^u_{ht} + (1 - \omega) \bar{r}_{ht} \]
\[ \bar{K}_m = \mathcal{H}_m \bar{L}_m \]
\[ \bar{Y}_m = Z_m \bar{K}_m^\alpha \bar{L}_m^{1-\alpha_m} \]
\[ \bar{k}_m^u = (1/\omega) \bar{K}_m \]
\[ \bar{x}_m^u = \delta_m \bar{k}_m^u \]
\[ \bar{c}_m^u = \left( \frac{\mu_m^u}{\mu_h^u} \right) \left( \bar{p}_{ht}^u \right)^{\varepsilon_h} \bar{c}_h^u \]
\[ \bar{c}_m^u = \left( \left( \frac{\mu_m^u}{\mu_h^u} \right) \left( \bar{p}_{ht}^u \right)^{\varepsilon_h} \bar{c}_h^u \right) \bar{c}_h^u \]
\[ \bar{\lambda}_m^u = \left( \bar{P}_m^u \right)^{-1} / \left[ \bar{c}_m^u - \left( \xi / (1 + \nu) \right) \left( \bar{l}^u_{ht} \right)^{1+\nu} \right] \]
\[ \tilde{\gamma}_m^u = \bar{p}_{ht}^u \bar{\lambda}_m^u \]
\[ \tilde{\eta}_m^u = \tilde{\lambda}_m^u \]
\[ \tilde{\eta}_m^u = \tilde{\lambda}_m^u \]

### 9.5 Commuting Costs and Relative Consumption Volatility

Here, we study the impact of changes in the commuting cost \( \tau \) on the relative volatility of consumption. For this, Appendix Figure 1 presents relevant quantities. Panel (a) shows the volatility of durable consumption purchases relative to the volatility of income for the constrained type-\( r \) household. Clearly, a reduction of the cost of commuting \( \tau \), a rise in \( 1 - \tau \), raises the relative volatility of durable consumption purchases. Here, durable expenditure is \( C_{dt}^r = (1 - \omega)x_{ht}^r \) while income is \( Y_r^r = (1 - \omega)(1 - \tau)W_0l_{mt} \). The reaction of the relative volatility to an increase in \( 1 - \tau \) has similarities to the reaction to an increase in \( \theta \). The relative volatility is an increasing and concave function of \( 1 - \tau \), but it does not display the hump shape observed for \( \theta \).

As for \( \theta \), changes in \( \tau \) affect the quantitative importance of the collateral constraint by affecting the deterministic steady state of the marginal product of home capital (the constrained asset) and
the deterministic steady state of the fraction of income that accrues to the constrained type-\( r \) households. The commuting cost parameter \( \tau \) affects the marginal product of capital via its effect on the capital-labor ratio. Recall that the capital-labor ratios in home production for both types of households are:

\[
\frac{k^r_h}{l^r_h} = \left( \frac{\alpha_h}{1 - \alpha_h} \right) \left[ \frac{\beta_r(1 - \tau)\bar{W}}{1 - \beta_r(1 - \delta_h) - \theta(\beta_u - \beta_r)} \right],
\]

\[
\frac{k^u_h}{l^u_h} = \left( \frac{\alpha_h}{1 - \alpha_h} \right) \left[ \frac{\beta_u\bar{W}}{1 - \beta_u(1 - \delta_h)} \right],
\]

where the market wage is given by \( \bar{W} = (1 - \alpha_m)Z_m \left[ \alpha_m Z_m/\tilde{R}_k \right]^{\alpha_m/(1-\alpha_m)} \) for a market marginal product of capital given by \( \tilde{R}_k = [1 - \beta_u(1 - \delta_m)] / \beta_u \). Obviously, the type-\( r \) home production capital-labor ratio is increasing in \( 1 - \tau \), but the type-\( u \) home production capital-labor ratio is invariant to \( \tau \). Thus, a reduction of \( \tau \), a rise in \( 1 - \tau \), raises the type-\( r \) capital-labor ratio and reduces the marginal product of home capital. Thus, as for \( \theta \), the rise in the capital-labor ratio works to reduce the productivity gap and lower the quantitative importance of the collateral constraint.

The parameter \( \tau \) also affects the deterministic steady state level of output and the fraction of output that accrues to the constrained type-\( r \) similarly to that of the loan-to-value parameter \( \theta \). Panel (b) documents that a rise in \( 1 - \tau \) raises total output, while panel (c) shows that a rise in \( 1 - \tau \) raises the income of type-\( r \) households relative to that of type-\( u \) households. Thus, a rise in \( \tau \) raises the steady state level and fraction of income that accrues to type-\( r \) households which raises the quantitative importance of the collateral constraint.

We note that the effects of \( \tau \) on the productivity gap and the fraction of income that accrues to type-\( r \) households are big, and that the trade-offs generate a concave relation, but not the hump shape.

### 9.6 Open Economy with Alternative Calibration

Following the literature, we verify the robustness of our results to different values for the parameter \( \chi \) that describes the sensitivity of the country premium to its debt level. For this exercise, we calibrate our model in the spirit of Garcia-Cicco et al. (2010) and set \( \chi = 1.0 \) and \( \tau = 1.2 \).

The standard business cycle moments predicted by the alternate calibration of our open economy model appear in Appendix Table 1, while the relative volatilities of consumption for different parametrizations appear in Appendix Figures 2 and 4. These results are quite similar to those in Table 2 and Figures 7 and 9. Overall, this confirms that our results are robust to the calibration of the parameter \( \chi \).
### Appendix Table 1
Second Moments for Alternate Open Economy Parametrization

<table>
<thead>
<tr>
<th></th>
<th>Volatility</th>
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<th>Volatility Relative to Output</th>
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</thead>
<tbody>
<tr>
<td>Output</td>
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<tr>
<td><strong>Output</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
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<tr>
<td><strong>Consumption</strong></td>
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<td></td>
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</tr>
<tr>
<td><strong>Nondurable</strong></td>
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<tr>
<td><strong>Durable</strong></td>
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<tr>
<td><strong>Investment</strong></td>
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<tr>
<td><strong>Net Export Ratio</strong></td>
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<tr>
<td><strong>Correlation with Output</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Nondurable</strong></td>
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</tr>
<tr>
<td><strong>Durable</strong></td>
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<td><strong>Investment</strong></td>
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<tr>
<td><strong>Net Export Ratio</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Autocorrelation</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Output</strong></td>
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</tr>
</tbody>
</table>

| **Net Export Ratio** | 0.71  |
| **Volatility**       | 3.02  |
| **Consumption**      | 1.15  |
| **Nondurable**       | 0.84  |
| **Durable**          | 4.60  |
| **Investment**       | 3.45  |
| **Net Export Ratio** | 0.71  |
| **Correlation with Output** |       | 0.95  |
| **Consumption**      | 0.97  |
| **Nondurable**       | 0.80  |
| **Durable**          | 0.83  |
| **Investment**       | 0.83  |
| **Net Export Ratio** | -0.51 |
| **Autocorrelation**  | 0.45  |
| **Output**           |        |
| **Net Export Ratio** | 0.10  |

Note: The numbers are the averages of 100 series of 500 periods simulated using the piecewise linear perturbation method of Guerrieri and Iacoviello (2015). Each relevant series is then detrended with the HP filter with a smoothness parameter of 100 except for the Net Exports Ratio.
Appendix Figure 1
Durable Volatility, Income, and Commuting Costs

Note: Panel (a) shows the ratio of the standard deviation of the cyclical component of durable consumption expenditures and the standard deviation of the cyclical component of income for type-r households against different values of the parameter $\tau$ that controls the level of commuting costs. Panels (b) and (c) show the deterministic steady state level of output and the deterministic steady state ratio of the income of type-r households to the income of type-u households for different values of $\tau$ that controls the level of commuting costs.
Appendix Figure 2
Financial Reforms and Consumption Volatility

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint and $\omega$ that controls the share of unconstrained type-$u$ households in the population.
Appendix Figure 3

Financial Reforms and Consumption Volatility: Type-\(r\) vs Type-\(u\)

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of income for different values of \(\theta\) that controls the tightness of the collateral constraint.
Appendix Figure 4
Crisis, Liberalization, and Consumption Volatility in the Open Economy

Note: The figure shows the ratio of the standard deviation of consumption and the standard deviation of output for different values of $\theta$ that controls the tightness of the collateral constraint and $\pi$ that controls the sensitivity of the country premium to expected future productivity.
Appendix Figure 5
The Relative Volatility of Consumption Expenditures

(a) Korea: Relative Volatility
(b) South Africa: Relative Volatility
(c) Korea: Output Volatility
(d) South Africa: Output Volatility

Note: Panels (a) and (b) show the ratio of the standard deviation of consumption (and components) to the standard deviation of output. Panels (c) and (d) show the standard deviation of output. The standard deviations are computed using 20-year rolling windows.
Appendix Figure 6
The Consumption Share of Durable Expenditures

<table>
<thead>
<tr>
<th>Year</th>
<th>SNA</th>
<th>No service</th>
<th>Dwellings</th>
<th>No service + dwellings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<tr>
<td>2015</td>
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</tbody>
</table>

Note: The figure shows the consumption share of durable expenditures for different notions of expenditures. “SNA” uses the definitions of the Systems of National Accounts; “No service” removes services from the definition of total consumption; “Dwellings” adds the component of investment devoted to residential construction or dwellings to durable expenditures and total consumption; and, finally, “No service + dwellings” removes services from total consumption and adds dwellings to both durable expenditures and total consumption.